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SHIPBOARD ANTENNA PATTERN INVERSION USING PRONY'S METHOD.(U)
SEP 78 J C LOGAN, J W ROCKWAY

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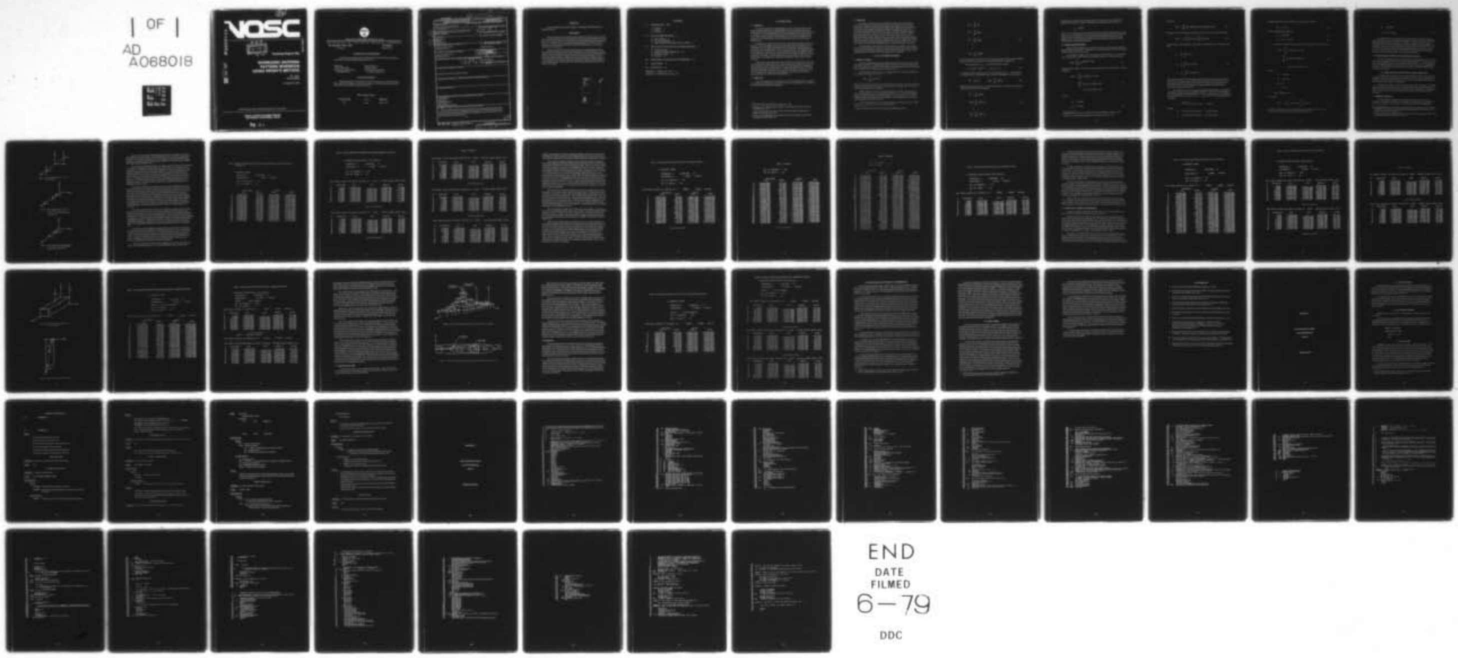

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Technical Report 362

SHIPBOARD ANTENNA PATTERN INVERSION USING PRONY'S METHOD

JC Logan
JW Rockway

30 September 1978

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Metric Conversion Table

To convert from	To	Multiply by
Feet	meters	3.048×10^{-1}

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Because far field antenna patterns can be expressed in terms of an exponential series of the spatial distribution of the source, they are candidates for Prony processing. Prony's method is used here to invert antenna pattern data to obtain the source distribution. Applications to pattern data characterization and pattern data compression are demonstrated.

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OBJECTIVE

Assess the feasibility, and then the utility, of inverting the far field patterns of shipboard antennas.

DISCUSSION

Prony processing may be applied to antenna patterns to determine source locations and can accurately identify the principal radiators even for the cluttered shipboard environment. The technique could be a very useful tool during the shipboard antenna arrangement design in cases where one or more parasitic structures are present. Not only can these structures be located by the Prony poles, but their relative importance can be deduced from the size of the residues.

Prony processing is not the only possible approach, as demonstrated via Fourier series processing. But the chief advantages of Prony are that (1) unlike the Fourier series approach, Prony processing can resolve sources separated by less than one-half wavelength, and (2) the solution is found with fewer iterations. Perhaps with experience or some suitable eigenvalue technique, as suggested by Lawrence Livermore Laboratory, only one or two iterations may be required. The only disadvantage of Prony is its susceptibility to noise in the pattern data. This problem is now receiving considerable attention from the technical community.

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1.0 INTRODUCTION

1.1 OVERVIEW

The far field pattern of an electromagnetic source is a Fourier transform of the spatial distribution of the source.[1] Because patterns can be expressed in terms of an exponential series of the spatial distribution, they are candidates for data transformation or "inversion." An inversion process which has been found to be increasingly useful is one based upon Prony's method. This procedure is one by which the coefficients and exponents of an exponential series can be derived from a sequence of equally spaced sampled values.

Data inversion could be useful in two ways: data characterization and data compression.[2] Prony processing can be used to characterize the source distribution of a far field pattern. Dr. E. K. Miller has indicated that the exponents in the spatial domain correspond to the physical locations of the sources in real space.[3] One application might be the imaging of a radiation source. Another possibility is that of developing a model for the object that in a perturbation sense could be used to predict its new behavior when its geometry is modified. A specific application for this capability would be the use of data obtained on the NOSC scale model range, to design and evaluate antenna systems on ships.[4]

The Prony's process might also be used to compress the original data. In this case, the coefficients and exponents may have no particular physical meaning, but would represent a minimum amount of numbers required to store and manipulate the original data. Thus ship's pattern data could be stored in a considerably reduced quantity, eg, instead of storing 360 complex numbers per pattern, it may be necessary to store only 4 to 10 pairs of complex Prony coefficients. This application could result in more efficient antenna optimization or direction finding algorithms.

1.2 OBJECTIVE

The technical objective of this study is to assess the feasibility and then the utility of inverting the far field patterns of shipboard antennas. Two applications are considered: data characterization and data compression.

---

1. Kraus, JD, Antennas, McGraw Hill Book Company, Inc., 1950.
2. Lawrence Livermore Laboratory Report, UCID-17511, Data Characterization and Compression, by EK Miller, July 6, 1977.
3. Lawrence Livermore Laboratory Report, UCID-1750s, Prony's Method for Angle Domain, by EK Miller and DL Lager, June 14, 1977.
4. Rockway, JW and DuBrul, DW, Performance Prediction Analysis for Shipboard Antenna Systems, Naval Engineers Journal, October 1977.

1.3 APPROACH

A number of alternatives to Prony processing of pattern data exist. However, evaluation and comparison of Prony-type processes is beyond the scope of this study, except where a similar or related technique might provide insight into the Prony process. An example is the evaluation of pattern data using a truncated Fourier series. This technique is considered along with the Prony method. Dr. Miller has suggested that the Fourier series approach is a special case of the more general Prony process.[5] Thus, the more familiar and extensive literature on the Fourier series can be used to provide insight into the Prony process. Both the Fourier series and the Prony methods are discussed in detail in section 2. Both methods have been implemented into a computer code called Angle Domain Pattern Analysis Program (ADPAP). This code is also discussed in section 2. A user's guide and code listing for ADPAP are given in appendices B and C respectively.

Two applications of antenna pattern data inversion are considered: data characterization and data compression. Section 3 presents the application to pattern data characterization. Section 4 presents the application to pattern data compression. In section 5, the feasibility and then the utility of pattern data inversion are summarized.

2.0 DATA INVERSION METHODS

2.1 PRONY'S METHOD

A ship is an electrically long narrow conducting platform with many definite antenna and parasitic protrusiles. A single transmitter driven antenna may excite one or more parasites. The result is an array which is nearly collinear. As a first approximation, the shipboard antenna problem will be treated using elementary array theory.

The far field of a collinear array of n isotropic point sources having coordinates X_α and complex amplitudes $\overline{A}_\alpha$ for $\alpha = 0, 1, 2, \ldots, n-1$ is

$$\overline{E}(\theta) = \sum_{\alpha=0}^{h-1} \overline{A}_\alpha \, e^{i\beta\cos\theta} \qquad (1)$$

where $\beta = 2\pi/\lambda$ and θ is the observation angle relative to the array axis. The pattern inversion problem is to determine the locations, X_α, and the comples amplitudes, $\overline{A}_\alpha$, of an array of sources which produce the far field $\overline{E}(\theta)$.

Prony's method is a technique for approximating a set of data as an exponential series. First the data must exist in equally spaced samples of $\cos\theta$. A linear interpolation scheme is sufficient to convert from equal spacing in θ to equal spacing in $\cos\theta$. Taking advantage of the symmetrical nature of the patterns to avoid multiple values of $\cos\theta$, the data is restricted to the interval $0 \leqslant \theta \leqslant \pi$.

Let δ be the increment of $\cos\theta$ for N samples and, for convenience, let $q_k = e^{i\beta x_k \delta}$ where $k = \alpha + 1$. Then (1) becomes a system of equations:

---

5. Lawrence Livermore Laboratory, Private Communication by EK Miller, June 1978.

$$\bar{E}_o = \sum_{k=1}^{n} \bar{A}_k$$

$$\bar{E}_1 = \sum_{k=1}^{n} \bar{A}_k q_k$$

$$\bar{E}_2 = \sum_{k=1}^{n} \bar{A}_k q_k \tag{2}$$

$$\vdots$$

$$\bar{E}_{n-1} = \sum_{k=1}^{n} \bar{A}_k q_k^{n-1}$$

If the q's were known, (2) would be a system of linear equations and the solution straightforward. However, in this case, the q's are unknown and (2) is nonlinear in q.

Next, assume that $q_1, 1_2 \ldots q_n$ are the roots of the equation:

$$q^n - a_1 q^{n-1} - a_2 q^{n-2} - \ldots - a_{n-1} q - a_n = 0 \tag{3}$$

Now multiply the first equation of (2) by $-\alpha_n$, the second by $-a_{n-1}, \ldots$ the (n−1)th by $-a_1$, and the nth by +1 and add the results. Using the relationship of (3), the result is:

$$\bar{E}_n = a_1 \bar{E}_{n-1} + \ldots\ldots + a_n \bar{E}_o = \sum_{k=1}^{n} a_k \bar{E}_{n-k} \tag{4}$$

Similarly, proceed starting with the second, third, fourth, etc., to the (N−n) th equation. The result is a linear system of equations:

$$\bar{E}_n = \sum_{k=1}^{n} a_k \bar{E}_{n-k}$$

$$\bar{E}_{n+1} = \sum_{k=1}^{n} a_k \bar{E}_{n+1-k} \tag{5}$$

$$\vdots$$

$$\bar{E}_{N-1} = \sum_{k=1}^{n} a_k \bar{E}_{N-1-k}$$

Equations (5) are linear in a and are easily solved. The values of q are then readily determined as the roots of (3). The source locations X_α, also called poles, are found from

$$q_k = e^{i\beta X_k \delta}$$

where $k = \alpha + 1$. Equation (2) can then be solved for the complex amplitudes $\overline{A}_k$, also called residues, of (1). Reference 6 gives a more complete discussion of Prony's method. Both the poles and residues are complex. The actual sources will have higher residues and purely imaginary poles. The other poles and residues which may appear are merely curve fitting values and have no physical interpretation.

2.2 FOURIER SERIES METHOD

Analysis of pattern data using a truncated Fourier series is an alternative approach to identifying the source locations.[7] The approach is somewhat simpler than Prony and provides different insight into pattern inversion.

The far field of a collinear array of n isotropic sources is given in (1). Let the amplitude be given by $\overline{A}_\alpha = a_\alpha e^{i\theta\alpha}$ where ϕ_α is the phase of the source α. Then (1) becomes:

$$\overline{E}(\theta) = \sum_{\alpha=0}^{n-1} a_\alpha e^{i(\beta X_\alpha \cos\theta + \phi_\alpha)} \qquad (6)$$

Noting that $E(\theta) = E_r(\theta) + i\, E_I(\theta)$, the real and imaginary parts of the field can be expanded separately:

$$E_r(\theta) = \sum_{\alpha=0}^{n-1} a_\alpha \cos(\beta X_\alpha \cos\theta + \theta_\alpha)$$

$$= \sum_{\alpha=0}^{n-1} [(a_\alpha \cos\theta_\alpha)\cos(\beta X_\alpha \cos\theta) \qquad (7)$$

$$-(a_\alpha \sin\theta_\alpha)\sin(\beta X_\alpha \cos\theta)]$$

Letting

$$I_\alpha = a_\alpha \cos\theta_\alpha$$

$$J_\alpha = -a \sin\theta_\alpha \qquad (8)$$

---

6. Hildebrand, FB, Introduction to Numerical Analysis, McGraw-Hill Book Company, Inc., 1956.

7. Kreyszig, E., Advanced Engineering Mathematics, John Wiley and Sons, Inc., 1962.

(7) becomes

$$E_r(\theta) = \sum_{\alpha=0}^{n-1} [I_\alpha \cos(\beta X_\alpha \cos\theta) + J_\alpha \sin(\beta X_\alpha \cos\theta)] \tag{9}$$

Assuming the sources are equally spaced, let $X_\alpha = \alpha d$ and $u = \beta d \cos\theta$, then (9) becomes:

$$E_r(\theta) = I_o + \sum_{\alpha=1}^{n-1} I_\alpha \cos(\alpha u) + \sum_{\alpha=1}^{n-1} J_\alpha \sin(\alpha u) \tag{10}$$

Equation (10) is a truncated Fourier series where the coefficients can be found in the usual way:

$$I_o = \frac{1}{2\pi} \int_{-\pi}^{\pi} E_r(u)\, du$$

$$I_\alpha = \frac{1}{\pi} \int_{-\pi}^{\pi} E_r(u) \cos(\alpha u)\, du \tag{11}$$

$$J_\alpha = \frac{1}{\pi} \int_{-\pi}^{\pi} E_r(u) \sin(\alpha u)\, du$$

The spacing of the sources and the angular spread of the pattern are critical to the evaluation of (11). Taking advantage of the symmetrical nature of the patterns, the pattern data may be restricted to the interval $0 \leqslant \theta \leqslant \pi$. The minimum source spacing for (11) to be valid is $d = \lambda/2$. This is the minimum separation between sources that can be physically resolved based on patterns.

The integrals in (11) can be evaluated using numerical integration. First subdivide the interval of integration into q equal parts of length $\delta u = \pi(\delta \cos\theta)$. The sampling of the pattern data must be in equal increments of $\cos\theta$ as in the Prony process. The sample points are located at $u_o = \pi \cos(o^\circ)$, $u_1, u_2, \ldots, u_q = \pi \cos(\pi)$. The first integral in (11) is found by:

$$I_o = \frac{(\delta \cos\theta)}{2} [E_r(u_o) + E_r(u_1) + \ldots . E_r(u_q)]$$

similarly

$$I_\alpha = (\delta \cos\theta)\,[E_r(u_o) \cos \alpha u_o + \ldots . E_r(u_q) \cos \alpha u_q]$$

$$J_\alpha = (\delta \cos\theta)\,[E_r(u_o) \sin \alpha u_o + \ldots . E_r(u_q) \sin \alpha u_q]$$

Associated with each pair of coefficients, (I_α, J_α) is a source location.

$$X_\alpha = \pm \alpha (\lambda/2) \tag{12}$$

The source amplitude and phase are

$$a_\alpha = (I_\alpha^2 + J_\alpha^2)^{1/2} \tag{13}$$

$$\phi_\alpha = \tan^{-1} (-J_\alpha/I_\alpha) \tag{14}$$

A similar set of equations can be developed for the imaginary part of the pattern.

$$E_I(\theta) = \sum_{\alpha=0}^{n-1} a_\alpha \sin (\beta X_\alpha \cos \theta + \phi_\alpha)$$

$$= \sum_{\alpha=0}^{n-1} [(a_\alpha \sin \phi_\alpha) \cos (\beta X_\alpha \cos \theta) \tag{15}$$

$$+ (a_\alpha \cos \phi_\alpha) \sin (\beta X_\alpha \cos \theta)]$$

Assume

$$I_\alpha = a_\alpha \sin \phi_\alpha$$

$$J_\alpha = a_\alpha \cos \phi_\alpha$$

$$X_\alpha = \alpha d$$

$$u = \pi \cos \theta$$

with $d = \dfrac{\lambda}{2}$ and $0 \leqslant \theta \leqslant \pi$.

The result is:

$$E_I(u) = I_o + \sum_{\alpha=1}^{n-1} I_\alpha \cos (\alpha u) + \sum_{\alpha=1}^{n-1} J_\alpha \sin (\alpha u) \tag{16}$$

where the coefficients, (I_α, J_α) are evaluated as before. Again, a source location is associated with each pair (I_α, J_α) with amplitude and phase of

$$a_\alpha = (I_\alpha^2 + J_\alpha^2)^{1/2}$$

$$\phi_\alpha = \tan^{-1} (I_\alpha/J_\alpha) \tag{17}$$

An ambiguity in the source locations exists if this approach is directly applied to pattern inversion. Equation (12) indicates the problem. Two source locations are actually associated with each pair of coefficients. What may occur is that two different arrays having different source locations and source strengths can produce identical patterns. The difficulty is easily averted by premultiplying the pattern data by a phase shift $e^{i\beta s}$, which in effect shifts the pattern phase center far enough along the assumed array axis so that the true source locations are revealed. Of course, this requires some prior knowledge of the spatial extent of the actual source so that the shift is sufficient.

2.3 ANGLE DOMAIN PATTERN ANALYSIS PROGRAM (ADPAP)

The Prony method of section 2.1 and the Fourier Series method of section 2.2 have been coded in the Angle Domain Pattern Analysis Program (ADPAP). A user's guide and program listing for ADPAP are given in appendices A and B, respectively. The Prony analysis portion of ADPAP is adapted from the Lawrence Livermore Laboratory code SEMPEX.[8] The Fourier series capability and shifting theorem coding were written at NOSC using the approach of section 2.2.

3.0 APPLICATION TO PATTERN DATA CHARACTERIZATION

The NOSC scale model pattern range has the capability to measure both magnitude and phase of the scale model antenna patterns. However, to date, pattern data with both magnitude and phase information does not exist. Also, time prohibited collection of such data for a wide range of antenna configurations.

Fortunately, a method of moments computer code, NEC[9], is available on the NOSC computer. NEC has equivalent or better accuracy than the scale model range and the versatility to model a wide range of antenna configurations in a timely manner. Consequently, NEC was used as the source of pattern data for the evaluation of the Prony and Fourier techniques.

3.1 SIMPLIFIED ARRAYS

The first problems considered were simple two- and three-element arrays. As confidence and insight were developed, more complex configurations were considered. A few of these elementary arrays (figures 1, 2, and 3) have been selected for illustration.

---

8. Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico, AFWL-TR-76-200, Users Manual for SEMPEX: A Computer Code for Extracting Complex Exponentials from a Time Wave Form, by DL Lager, HG Hudson, and AJ Poggio, 1976

9. Naval Ocean Systems Center Technical Document 116, Numerical Electromagnetic Code (NEC) – Method of Moments, by GJ Burke and AJ Poggio, 18 July 1977.

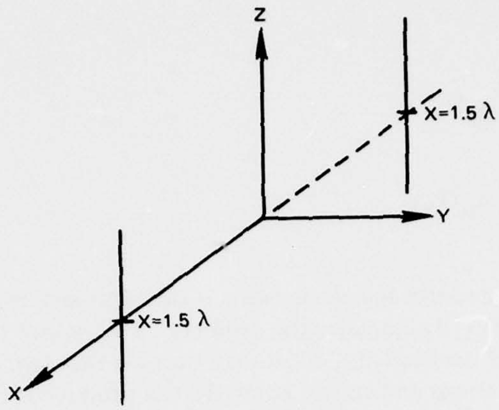

Figure 1. Two half-wave dipoles at X = ± 1.5 λ driven in phase.

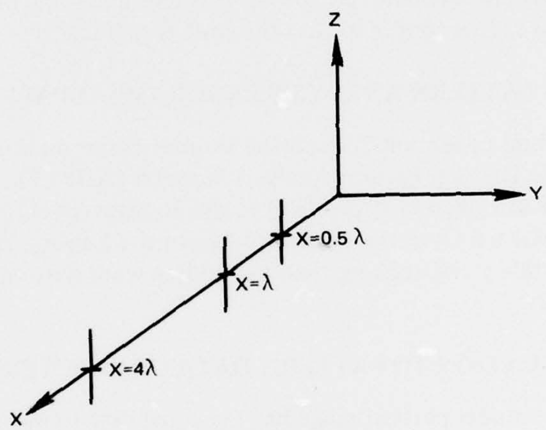

Figure 2. Three half-wave dipoles at X = 0.5 λ, λ, and 4 λ. The dipole at X = λ is excited 180° out of phase with respect to the other two.

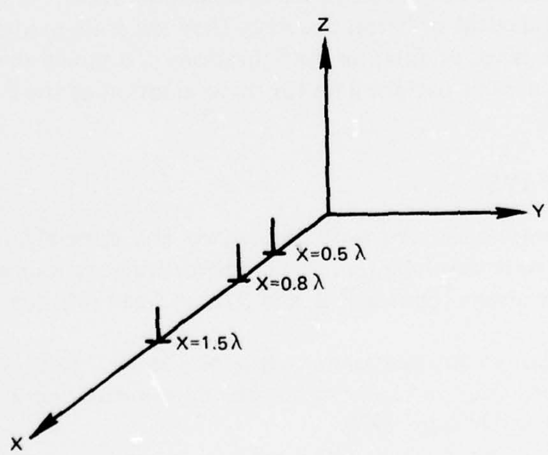

Figure 3. Three quarter-wave monopoles located at X = 0.5 λ, 0.8 λ, and 1.5 λ. Only the dipole at X = 0.5 λ is driven.

Figure 1 shows two half-wave dipoles separated by three wavelengths in free space. The dipoles are center-fed in phase. The far field pattern in the X–Y plane, the magnitude and phase of the vertical component of the electric field, was calculated using NEC. Then the Prony process and Fourier series analysis were applied to the pattern data using ADPAP. The results are shown in tables 1 and 2.

The residues and poles listed in table 1 are related to the source patterns and locations by equation (1). Residues are the amplitudes, $\overline{A}_{\alpha}$, and the poles are the arguments of the exponentials. The residues correspond to the magnitude and phase of possible sources, and the poles correspond to the source locations in meters in a complex spatial domain. Real sources have poles on the real axis. In this case, pole numbers 1 and 2 have the highest relative residues and are therefore prime source candidates. The real parts of these poles are nearly zero compared to the other poles in the list. The imaginary part gives the source location as ± 1.5 meters along the array axis. Note that the source strengths are the same magnitude and phase. The other poles listed appear as curve fitting poles and have no relationship to physical sources.

The solution of the matrices in the Prony process used by ADPAP is presently set up for square matrices. This requires that the number of pattern data samples be twice the number of poles specified. Errors are introduced by specifying too many or too few poles, either because of insufficient data samples to characterize the pattern or because overspecification of poles introduces curve fitting noise. Without some a priori knowledge of the number of poles, and hence data samples required, the Prony process must be applied repeatedly until a stable solution can be obtained. Of course this requires some knowledge of the source distribution in the first place, but this is an existing condition for this particular application. Table 1 is the result of several iterations of the Prony process applied to the pattern data by specifying a different number of poles (eg, number of poles = 10, 15, 20, 25, . . .).

The Fourier coefficients listed in table 2 are related to the pattern data of the array in figure 1 by equation (10). They were found by expanding the real component of the electric field in a Fourier series as discussed in section 2.2. The source strength and location associated with each pair of Fourier coefficients, I(N) and J(N), are given by equations (12), (13), and (14). The values of location and strength are listed in the columns under S, AMP (N), and PHASE, respectively. A different set of Fourier coefficients could also be obtained by expanding the imaginary component but the source strengths and locations are identical. Hence the examples discussed will use the Fourier series expansion of the real component only.

One of the options of ADPAP is to apply a shift of exp $(i\beta X \cos\theta)$ to the pattern in n equal steps prior to computation of the Fourier coefficients. In effect, if the observer is on the array axis (the X axis in figure 1), the shift moves the phase center away from the observer. The source locations, S, in table 2 are given with respect to the original phase center. Five sets of coefficients are shown for varying amounts of shift from zero to 2.0 meters. Originally the shift was applied in equal increments of 0.1 meters, but in the interest of conserving space, only five of the steps are illustrated.

The Fourier coefficients, and the source amplitudes as a result, rise and fall as the phase center is shifted along the array axis. Larger amplitudes indicate possible source

Table 1. Residues and poles from Prony processing of the pattern of two half-wave dipoles. (See figure 1.)

```
---COMPLEX PRONY
    FREQUENCY =          .2998+009     HZ
    WAVELENGTH =              1.0000     METERS
    NO. OF SAMPLES =     40
    NO. OF POLES =     20
```

	RESIDUES MAG	PHASE	POLES REAL	IMAG
1	.59669+002	89.794	.86443-003	.15006+001
2	.59676+002	90.205	.86118-003	-.15006+001
3	.12962+001	94.039	-.10942+001	-.82827+001
4	.10098+001	-41.821	-.13697+001	.61389+001
5	.23319+001	129.813	-.10802+001	-.19335+001
6	.28862+001	-43.000	-.10993+001	.54456+000
7	.14589+001	-157.482	-.13228+001	.41318+001
8	.10373+001	-83.162	-.14082+001	-.51536+001
9	.11945+001	147.886	-.97014+000	.93118+001
10	.11928+001	32.136	-.96988+000	-.93116+001
11	.28861+001	-137.003	-.10991+001	-.54518+000
12	.23328+001	50.163	-.10798+001	.19325+001
13	.19086+001	56.178	-.12363+001	-.30200+001
14	.19105+001	123.833	-.12360+001	.30194+001
15	.14580+001	-22.495	-.13228+001	-.41324+001
16	.10383+001	-96.833	-.14081+001	.51528+001
17	.10084+001	-138.131	-.13700+001	-.61395+001
18	.11563+001	18.633	-.12416+001	.71772+001
19	.11551+001	161.450	-.12417+001	-.71775+001
20	.12974+001	86.051	-.10944+001	.82825+001

Table 2. Fourier coefficients for the pattern of two half-wave dipoles. (see figure 1.)

---FOURIER COEFFICIENTS FOR REAL(E)

FREQUENCY = .2998+009 HZ

WAVELENGTH = 1.0000 METERS

NO. OF SAMPLES = 100

NO. OF TERMS = 10

THE PHASE CENTER HAS BEEN SHIFTED BY .0000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	.0000	-.51945-004		.12159+001	-90.002
1	.5000	.16184-005	.30897-011	.16184-005	-.000
2	1.0000	-.16706-005	.54534-012	.16706-005	-180.000
3	1.5000	.16205-005	.11826-011	.16205-005	.000
4	2.0000	-.15867-005	-.99932-012	.15867-005	180.000
5	2.5000	.16922-005	.63953-011	.16922-005	-.000
6	3.0000	-.35558-004	.61097-011	.35558-004	-180.000
7	3.5000	.16931-005	.14584-011	.16931-005	.000
8	4.0000	-.15937-005	-.55356-011	.15937-005	180.000
9	4.5000	.16519-005	.22368-011	.16519-005	.000
10	5.0000	-.16653-005	-.48625-011	.16653-005	180.000

****

THE PHASE CENTER HAS BEEN SHIFTED BY .5000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-.5000	-.15292-006		.12936+001	90.000
1	.0000	-.51234-004	-.20931-001	.20931-001	90.140
2	.5000	.93947-005	.59906+002	.62630+002	-90.000
3	1.0000	.35882-006	-.71980-002	.71987-002	89.997
4	1.5000	-.75523-005	-.59904+002	.56734+002	90.000
5	2.0000	-.20108-004	.28568-001	.28568-001	-90.040
6	2.5000	.14919-005	-.53879-002	.53881-002	89.984
7	3.0000	-.16921-004	.40430-003	.40465-003	-92.397
8	3.5000	.12258-005	-.13001-002	.13001-002	89.946
9	4.0000	-.10633-005	.21349-002	.21348-002	-90.029
10	4.5000	.71527-006	-.64536-002	.64534-002	89.994

****

Table 2. Continued.

THE PHASE CENTER HAS BEEN SHIFTED BY 1.5000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-1.5000	.75740-005		.63011+002	90.000
1	-1.0000	.25672-005	-.71737-002	.71736-002	89.979
2	-.5000	-.14016-005	.17047-002	.17047-002	-90.047
3	.0000	-.68424-004	.44357-003	.44882-003	-98.769
4	.5000	.11080-006	-.36705-002	.36701-002	89.998
5	1.0000	.39012-006	.21765-001	.21761-001	-89.999
6	1.5000	-.80195-005	-.59911+002	.60243+002	90.000
7	2.0000	-.13342-005	.31119-001	.31122-001	-90.002
8	2.5000	-.39975-006	-.13150-001	.13151-001	90.002
9	3.0000	-.12080-004	-.22106-002	.22106-002	90.313
10	3.5000	-.53943-006	-.11006-001	.11007-001	90.003

****

THE PHASE CENTER HAS BEEN SHIFTED BY 1.6000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-1.6000	.38147-005		.62952+002	90.000
1	-1.1000	.93055-005	-.22748+002	.23072+002	90.000
2	-.6000	-.66661-005	.10004+002	.99559+001	-90.000
3	-.1000	-.54735-004	-.51830+001	.51813+001	90.001
4	.4000	-.18804-004	.15618+001	.15619+001	-90.001
5	.9000	.14527-004	.39396+001	.39458+001	-90.000
6	1.4000	-.18977-004	-.51470+002	.51077+002	90.000
7	1.9000	.63193-005	-.17939+002	.17621+002	90.000
8	2.4000	-.78799-005	.96998+001	.96263+001	-90.000
9	2.9000	-.26973-006	-.71088+001	.61706+001	90.000
10	3.4000	-.15875-004	.57281+001	.57299+001	-90.000

****

THE PHASE CENTER HAS BEEN SHIFTED BY 2.0000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-2.0000	.15991-005		.12469+001	-90.000
1	-1.5000	.65409-005	-.59904+002	.62117+002	90.000
2	-1.0000	-.15803-004	.21390-001	.21390-001	-90.042
3	-.5000	-.92475-006	-.36809-002	.36808-002	90.014
4	.0000	-.49394-004	.84596-003	.84741-003	-93.342
5	.5000	-.62612-006	-.49693-002	.49691-002	90.007
6	1.0000	-.62618-006	.23892-001	.23891-001	-90.002
7	1.5000	-.10274-004	-.59917+002	.57775+002	90.000
8	2.0000	.39399-005	.26372-001	.26371-001	-89.991
9	2.5000	-.35158-006	-.16399-001	.16397-001	90.001
10	3.0000	-.16121-004	-.11239-001	.11240-001	90.082

locations. By stepping in fractional wavelengths through a total shift of at least one-half wavelength, the source locations can be determined within an error equal to the step size. A maximum in amplitude occurs when the coefficients coincide with real source locations.

The first set of coefficients in table 2 are the results for zero shift. The first coefficient has a significant amplitude compared to the amplitudes of the other coefficients indicating that a single source could be located at the origin. However, as a shift is applied to the pattern data in 0.1 increments, the amplitudes of some of the other coefficients grow indicating different possible source locations. With a shift of 0.5 meters (second set of coefficients in table 2), three possible source locations are indicated at –0.5, + 0.5 and + 1.5 meters along the X axis. As the shift is applied in small steps (0.1λ or less), the possible source locations appear to move around. When the shift is sufficient to encompass the real source locations, the amplitudes predicted from the Fourier coefficients reach a maximum. In this case, an applied shift of 1.5 meters (third set of coefficients) correctly predicts the source locations at ± 1.5 meters. If the incremental shifting is continued, the amplitudes will peak every time they coincide with the true source locations. For example, when the shift is 2 meters (last set of coefficients), the true source locations are correctly given by the peak amplitudes.

The Fourier series expansion assumes an array of sources equally spaced at a half-wavelength. It is therefore possible that the array sources may not exactly line up with the true source locations at each increment of the applied shift, even though the shift may encompass the entire source distribution. This condition is illustrated in table 2 for a shift of 1.6 meters (fourth set of coefficients). A peak in amplitudes still occurs near the actual source locations (± 1.5 meters), but other coefficients have significant amplitudes as well. Notwithstanding, the maximum amplitudes occur when the coefficients align with true source locations and, again, they are clearly defined by a significant difference in amplitude compared to the other members of the set.

The next example to consider is that of the array shown in figure 2. In this case, the sources are all located along the positive X axis. Figure 2 shows three half-wave dipoles located at $X = 0.5\lambda$, λ, and 4λ in free space. All three are center-fed with the dipole at $X = \lambda$ driven 180° out of phase. The pattern in the X-Y plane was calculated using NEC and processed using the Prony and Fourier options of ADPAP. These results are given in tables 3 and 4.

Table 3 gives the Prony residues and poles for the cases of 20, 30, and 40 poles. The number of samples in pattern data are consequently 40, 60, and 80 respectively. For 20 poles (the first set of poles in table 3), four possible source locations (pole numbers 1, 3, 6, and 11) are identified by their relatively large residues. These poles are located on the imaginary axis (small real parts) of the complex spatial plane corresponding to the correct source locations along the array axis (see the imaginary part of the poles). A double pole is located at 3.9816 meters (poles 6 and 11). By increasing the number of poles to 30 (see second set in table 3), the double-pole ambiguity is resolved and the correct source locations indicated by poles 1, 2, and 17. Increasing the number of poles to 40 (third set in table 3) reintroduces the double poles (pole numbers 7 and 22). The double poles result from curve fitting.

Table 3. Prony residues and poles for the pattern of the dipoles in figure 2.

```
---COMPLEX PRONY

    FREQUENCY =          .2998+009     HZ
    WAVELENGTH =             1.0000      METERS
    NO. OF SAMPLES =    40
    NO. OF POLES =    20

THE PHASE CENTER IS LOCATED AT (      .0000,       .0000)    METERS
```

	RESIDUES		POLES	
	MAG	PHASE	REAL	IMAG
1	.61641+000	90.543	.16647-004	.49738+000
2	.13337-002	89.651	.16835+000	-.89905+000
3	.84378+000	144.120	-.22716-002	.99842+000
4	.57368-002	106.658	.13713+000	-.29011+001
5	.29082-001	140.166	-.22141-001	-.46901+001
6	.34433+000	70.105	.74874-001	.39816+001
7	.26785-002	-12.507	.18799-001	-.74497+001
8	.40087-002	-173.721	-.18044+000	-.16822+001
9	.51792-002	159.586	-.45964-001	.25687+001
10	.22496-007	-167.040	.24731+001	-.61194+001
11	.34459+000	44.973	-.69454-001	.39815+001
12	.26857-002	-147.478	-.97249-001	-.37497+001
13	.93247-003	126.637	.57388+000	.84014+001
14	.21924-004	-130.260	.11939+001	.62213+001
15	.58713-003	19.881	-.55092+000	.83520+001
16	.37379-001	-51.529	.52114-002	-.58396+001
17	.51041-004	-126.352	-.98006+000	.62740+001
18	.45116-002	5.310	.23334+000	-.93319+001
19	.28459-002	-61.081	-.17880+000	-.89795+001
20	.23647-015	124.195	-.51741+001	-.47773+001

****

Table 3. Continued.

NO. OF SAMPLES = 60

NO. OF POLES = 30

	RESIDUES MAG	PHASE	POLES REAL	IMAG
1	.63641+000	87.565	.35830-002	.50659+000
2	.84183+000	144.866	.41111-002	.99975+000
3	.17543-005	84.814	.20555+001	-.34274+001
4	.11646-004	17.491	.16739+001	-.13498+001
5	.85939-003	-99.899	.85461+000	.32198+001
6	.39956-001	-146.791	-.11567-001	.44217+001
7	.59911-003	143.269	.91706+000	.86623+001
8	.58664-004	124.645	.13957+001	.10589+002
9	.76701-006	164.578	.20651+001	-.55086+001
10	.33275-004	82.760	.14895+001	.12552+002
11	.19747-003	54.174	.11161+001	.14562+002
12	.92178-002	140.899	.22092+000	.67071+001
13	.15404-006	102.436	.20092+001	-.99438+001
14	.26894-016	76.108	.62029+001	-.18997+001
15	.40842-005	166.614	.17667+001	-.11880+002
16	.18212-006	-86.098	.18926+001	-.78345+001
17	.66413+000	58.044	-.20902-002	.40127+001
18	.46419-002	137.201	.39599+000	-.13776+002
19	.32826-004	28.971	-.13939+001	-.15850+001
20	.12800-005	53.448	-.20472+001	-.50045+001
21	.57901-003	91.927	-.79053+000	.26103+001
22	.28726-005	-12.257	-.17059+001	-.90084+001
23	.97818-003	102.545	-.40408+000	.75901+001
24	.39791-005	45.008	-.18667+001	-.32972+001
25	.11036-003	101.513	-.94000+000	.96369+001
26	.15413-005	58.299	-.18594+001	-.67752+001
27	.46884-004	149.681	-.11360+001	.11591+002
28	.49104-005	-66.381	-.16237+001	-.10944+002
29	.22615-004	-123.594	-.10594+001	-.12866+002
30	.38708-004	-128.961	-.10580+001	.13677+002

****

Table 3. Continued.

NO. OF SAMPLES = 80

NO. OF POLES = 40

	RESIDUES MAG	PHASE	POLES REAL	IMAG
1	.55917+000	83.165	-.94046-003	.47421+000
2	.46751-002	109.704	.51609+000	.45050+000
3	.87042+000	140.956	-.11839-001	.99447+000
4	.14351-002	-116.957	-.43422-002	-.10519+001
5	.42155-002	57.719	.19453-001	-.44592+001
6	.18926-002	-135.033	.65352-001	-.74896+001
7	.32231+000	74.764	.61469-001	.39837+001
8	.49213-002	-118.058	.40063-001	-.67199+001
9	.92183-004	-67.995	.75367+000	-.98276+001
10	.27721-002	-122.170	-.57337-001	-.14606+002
11	.29289-002	54.977	-.39743-001	-.17033+002
12	.20791-003	-128.436	.65320+000	.60126+001
13	.18055-002	40.931	.34943-001	-.16077+002
14	.52790-003	144.444	.54654+000	.80318+001
15	.10264-002	-128.730	-.18390-001	-.33162+001
16	.19824-002	-127.441	-.29642-001	.93306+001
17	.29968-002	56.871	.67545-001	.10161+002
18	.60731-002	53.562	-.15284-001	.12480+002
19	.63080-002	170.132	-.50248+000	.76226+000
20	.19088-002	57.410	-.26144-001	-.22018+001
21	.99954-002	-123.481	.17246-001	.13943+002
22	.38270+000	44.394	-.51451-001	.39852+001
23	.88759-003	-43.076	-.24952-001	-.91836+001
24	.29941-002	-104.225	.43371-002	.11326+002
25	.11214-002	46.290	.73469-001	-.10859+002
26	.70546-003	-67.525	-.49794+000	.78970+001
27	.62460-002	-118.050	.26747-001	-.59463+001
28	.34150-002	-120.731	.32571-001	.14755+002
29	.31071-002	-119.663	.28984-001	.15645+002
30	.10730-002	-122.040	-.35307-002	-.12093+002
31	.24942-003	-106.997	.17671+000	.17520+002
32	.84597-007	-132.630	.20745+001	.15490+002
33	.18427-003	-104.607	-.72630+000	.60203+001
34	.15908-002	-127.215	-.15946+000	-.15512+002
35	.12747-002	52.650	.10211-001	.18784+002
36	.10865-009	7.453	-.30827+001	-.93130+001
37	.11563-002	-136.373	-.24977-001	-.13969+002
38	.14341-003	-128.300	-.56386+000	-.18566+002
39	.10417-002	65.033	.13504+000	-.19393+002
40	.16170-008	105.363	-.27291+001	.17265+002

Table 4. Fourier coefficients for the pattern of the dipoles in figure 2.

```
---FOURIER COEFFICIENTS FOR REAL(E)

    FREQUENCY =         .2998+009    HZ

    WAVELENGTH =           1.0000     METERS

    NO. OF SAMPLES =    100

    NO. OF TERMS =    10

THE PHASE CENTER IS LOCATED AT (      .0000,      .0000)   METERS
```

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	.0000	-.32179-002		.57129-002	124.282
1	.5000	.22666-001	-.61738+000	.61780+000	87.897
2	1.0000	-.68859+000	-.49685+000	.84912+000	144.188
3	1.5000	.65925-002	.18012-002	.68341-002	-15.281
4	2.0000	-.62107-002	-.14291-002	.63730-002	167.041
5	2.5000	.55753-002	.65068-003	.56131-002	-6.657
6	3.0000	-.51805-002	-.19898-002	.55495-002	158.989
7	3.5000	.12543-002	.61751-002	.63012-002	-78.518
8	4.0000	.35906+000	-.59092+000	.69145+000	58.716
9	4.5000	.89243-003	.74859-002	.75390-002	-83.202
10	5.0000	-.55128-002	-.36989-002	.66387-002	146.139

Table 4 lists the Fourier coefficients for the pattern of the array in figure 2. The dipoles are all nicely arranged on one side of the phase center at integer multiples of $\lambda/2$. Consequently, only one iteration of the Fourier series is really necessary to identify the sources. Coefficients 1, 2, and 8 in table 4 have the largest magnitudes and are sharply defined. It is interesting to note that the amplitude and phase of each source predicted by Fourier is essentially the same as those predicted by Prony (see the second set of poles in table 3).

The two examples thus far have been nicely arranged, ie, the dipoles have had integer multiples of $\lambda/2$ spacing. Figure 3 shows an array in which two of the monopoles are 0.3λ apart. The monopoles are all located on one side of the phase center as in the previous case. For this example, the patterns were calculated for the case with only one monopole excited: the one at $X = 0.5\lambda$. In this case, the monopole at 1.5λ is too weakly coupled to contribute much to the pattern. The results of applying the Prony and Fourier processing are given in tables 5 and 6.

Table 5 shows the Prony residues and poles for the array of monopoles in figure 3. After a number of iterations using a different number of poles each time, it was found that about 30 poles with 60 sample points is satisfactory. Two poles are distinctly identified by their relatively larger residues (pole numbers 2 and 3). They are located on the imaginary axis of the complex spatial plane as required. Pole number 2 has a greater residue than pole number 3, which should be expected since it corresponds to the driven source at 0.5 meters on the array axis. The Prony process has no difficulty in correctly resolving the source locations even though they are separated by only 0.3λ.

Table 6 shows four sucessive iterations of the Fourier process in which the phase center is shifted 0.1 meter along the array axis each time. The amplitudes peak around the proper locations for two sources at $X = 0.5\lambda$ and 0.8λ. However, the true locations are not readily apparent. The Fourier process cannot resolve sources that are closer than 0.5λ apart.

3.2 ANTENNAS IN A SIMPLE ENVIRONMENT

The previous three examples were for simple arrays. The only radiating structures were the dipoles, and they were arranged along a common axis.

Figure 4 shows an array of 35-foot whips mounted on a box whose dimensions are similar to the superstructure of a small ship. The 35-foot whips are 8.69 meters apart and form an array whose axis intercepts the X-Z plane at 164.75° (see figure 5). At hf frequencies where these antennas are employed, currents, which also radiate and influence the patterns, are induced on the box surface. For this illustration, the whip at the box center is driven at 20 MHz so the whips are 0.58λ apart. Both Prony and Fourier should be able to resolve the true sources. The whips and the box were modeled using NEC and the resulting pattern processed using ADPAP. Some of these results are listed in tables 7 and 8.

The patterns calculated by NEC were for the phase center at the origin as shown in figure 4. When Prony was applied, the resulting matrices were ill-conditioned, causing an abnormal abort of the program. This problem was observed even with the simple array problems previously discussed when the assumed array axis did not coincide with the true axis of the array. If the assumed axis is off 5 to 6 degrees, acceptable results may be obtained.

Table 5. Prony residues and poles for the pattern of the array in figure 3.

---COMPLEX PRONY

FREQUENCY = .2998+009 HZ

WAVELENGTH = 1.0000 METERS

NO. OF SAMPLES = 60

NO. OF POLES = 30

THE PHASE CENTER IS LOCATED AT (.0000, .0000) METERS

	RESIDUES		POLES	
	MAG	PHASE	REAL	IMAG
1	.42136-003	92.977	-.35895-001	-.13346+000
2	.10167+001	52.877	-.16557-002	.50336+000
3	.50206+000	152.947	.92165-002	.80206+000
4	.24765-004	114.411	.53449+000	-.16028+001
5	.33125-004	-153.740	.43943+000	.35144+001
6	.10081-005	-30.790	.10012+001	.79483+001
7	.16913-004	-25.596	.60108+000	-.43460+001
8	.89118-001	-122.082	.19033-002	.14921+001
9	.56485-021	-90.995	.69601+001	-.76495+001
10	.13287-004	-36.419	.58484+000	.12679+002
11	.57516-005	-143.455	.85356+000	-.70479+001
12	.17603-004	-53.222	-.76860+000	-.37654+001
13	.12042-005	-51.646	.11626+001	-.91657+001
14	.40049-004	14.728	.26872+000	.59783+001
15	.48042-004	88.848	-.32948+000	-.11095+002
16	.44766-005	-121.928	.75230+000	-.12900+002
17	.57594-004	-100.471	-.49487+000	-.20345+001
18	.50265-004	29.142	-.50913+000	.39355+001
19	.14092-004	33.361	-.82421+000	.77080+001
20	.79368-005	-30.999	-.86774+000	-.72533+001
21	.64959-003	-32.634	-.12842+000	.97510+001
22	.19237-003	-158.660	-.45028+000	.99775+001
23	.13309-004	-42.618	-.85969+000	-.55038+001
24	.38634-004	16.165	-.58138+000	.58294+001
25	.65818-005	48.248	-.77152+000	.11343+002
26	.14786-004	-72.243	-.67832+000	.14463+002
27	.15759-003	-35.370	.91925-001	.14376+002
28	.38737-006	2.161	-.11480+001	-.91085+001
29	.10872-004	90.628	-.75441+000	-.13308+002
30	.18148-011	11.255	-.34657+001	-.11840+002

Table 6. Fourier coefficients for the pattern of the array in figure 3.

---FOURIER COEFFICIENTS FOR REAL(E)

FREQUENCY = .2998+009 HZ

WAVELENGTH = 1.0000 METERS

NO. OF SAMPLES = 100

NO. OF TERMS = 10

THE PHASE CENTER HAS BEEN SHIFTED BY .0000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	.0000	.76633-001		.93782-001	-35.200
1	.5000	.34787+000	-.88524+000	.95114+000	68.547
2	1.0000	-.31175+000	-.21099+000	.37644+000	145.910
3	1.5000	.34032-001	.14416+000	.14812+000	-76.717
4	2.0000	-.46162-001	-.45243-001	.64636-001	135.576
5	2.5000	.33406-001	.33733-001	.47475-001	-45.279
6	3.0000	-.27106-001	-.27066-001	.38306-001	135.043
7	3.5000	.23517-001	.22632-001	.32638-001	-43.902
8	4.0000	-.21260-001	-.19483-001	.28838-001	137.497
9	4.5000	.19725-001	.17085-001	.26095-001	-40.898
10	5.0000	-.18637-001	-.15131-001	.24005-001	140.928

****

THE PHASE CENTER HAS BEEN SHIFTED BY .1000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-.1000	-.54365-001		.15809+000	-110.113
1	.4000	.49828+000	-.70895+000	.86654+000	54.899
2	.9000	-.29279+000	-.48613+000	.56749+000	121.060
3	1.4000	-.13045-001	.23215+000	.23252+000	-93.216
4	1.9000	-.30836-001	-.91075-001	.96154-001	108.705
5	2.4000	.22440-001	.74314-001	.77628-001	-73.197
6	2.9000	-.19879-001	-.61337-001	.64478-001	107.958
7	3.4000	.18710-001	.52018-001	.55280-001	-70.217
8	3.9000	-.18062-001	-.45100-001	.48582-001	111.825
9	4.4000	.17641-001	.39726-001	.43467-001	-66.055
10	4.9000	-.17357-001	-.35368-001	.39397-001	116.139

****

Table 6. Continued.

THE PHASE CENTER HAS BEEN SHIFTED BY .2000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-.2000	-.13361+000		.21112+000	-129.261
1	.3000	.54051+000	-.49263+000	.73132+000	42.346
2	.8000	-.18921+000	-.73023+000	.75435+000	104.526
3	1.3000	-.98411-001	.25508+000	.27340+000	-111.097
4	1.8000	.26219-002	-.95415-001	.95451-001	88.426
5	2.3000	-.27821-002	.86249-001	.86294-001	-91.848
6	2.8000	-.11809-002	-.72319-001	.72329-001	90.936
7	3.3000	.39743-002	.61686-001	.61813-001	-86.314
8	3.8000	-.58690-002	-.53621-001	.53941-001	96.246
9	4.3000	.71759-002	.47300-001	.47841-001	-81.373
10	4.8000	-.81162-002	-.42167-001	.42941-001	100.895

****

THE PHASE CENTER HAS BEEN SHIFTED BY .3000 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-.3000	-.15062+000		.19083+000	-142.120
1	.2000	.46774+000	-.27750+000	.54387+000	30.680
2	.7000	-.16414-001	-.89593+000	.89608+000	91.050
3	1.2000	-.20120+000	.18792+000	.27531+000	-136.954
4	1.7000	.39401-001	-.49406-001	.63194-001	51.428
5	2.2000	-.32307-001	.63855-001	.71563-001	-116.837
6	2.7000	.21383-001	-.55203-001	.59200-001	68.826
7	3.2000	-.14630-001	.47559-001	.49759-001	-107.099
8	3.7000	.10290-001	-.41534-001	.42789-001	76.085
9	4.2000	-.73599-002	.36733-001	.37463-001	-101.330
10	4.7000	.52885-002	-.32798-001	.33222-001	80.840

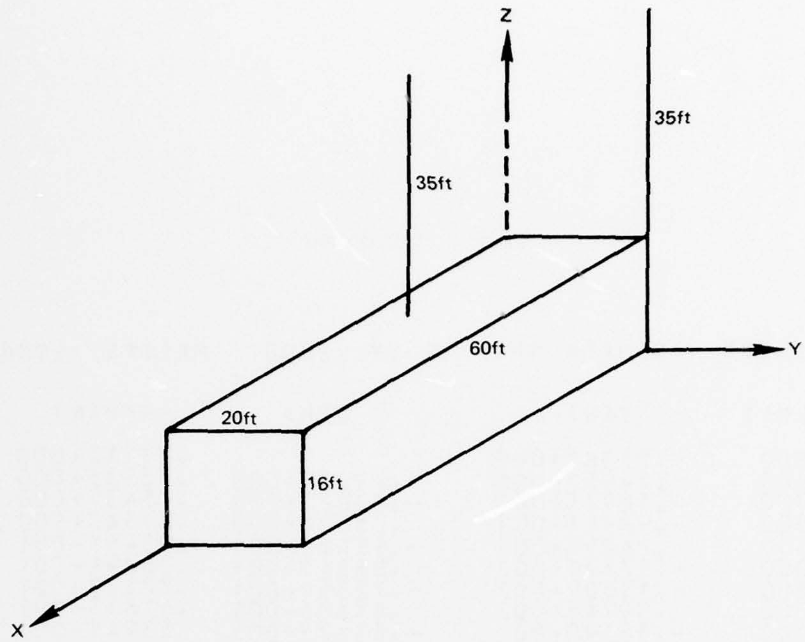

Figure 4. Two 35-foot whips mounted on a box over perfect ground.

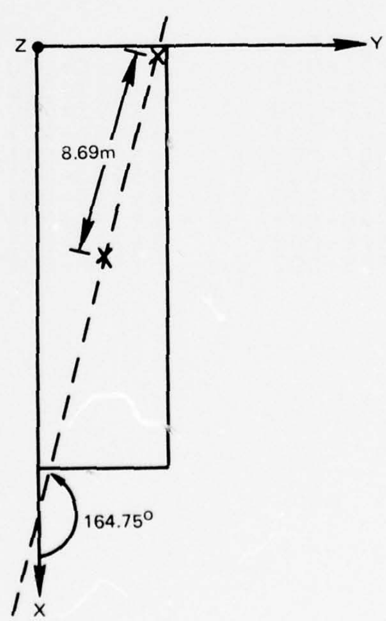

Figure 5. Top view of the configuration of figure 4.

Table 7. Prony residues and poles for the pattern produced by the configuration in figure 4.

---COMPLEX PRONY

FREQUENCY = .2000+008 HZ

WAVELENGTH = 14.9900 METERS

START = 165.000 DEGREES

NO. OF SAMPLES = 50

NO. OF POLES = 25

THE PHASE CENTER IS LOCATED AT (9.1440, 3.0480) METERS

	RESIDUES		POLES	
	MAG	PHASE	REAL	IMAG
1	.25221+002	-16.960	-.53407+000	.18749-001
2	.29126+001	-29.669	-.10920+001	-.84258+001
3	.82630+001	30.863	-.29286+000	.84185+001
4	.55381-003	13.573	.87853+001	-.26556+002
5	.53054-002	-106.777	.48710+001	.69715+002
6	.35237-002	60.984	.83898+001	-.11147+003
7	.86535-003	-89.642	.99448+001	-.14726+003
8	.87582-003	-79.446	.17931+003	.20237+002
9	.13989-001	47.302	-.26526+001	.18104+003
10	.46118-002	-47.599	.46636+001	.13505+003
11	.21204-001	-95.734	-.16040+001	-.99451+002
12	.89169-006	113.407	.34350+002	-.98573+001
13	.76627-003	-133.884	.11645+001	.39464+002
14	.25227-001	-34.403	-.92421-001	.77511+002
15	.40311-001	34.755	-.86290-001	.10205+003
16	.19416-001	-161.216	.80439+000	.11738+003
17	.71297-002	-55.854	-.16885+001	.15175+003
18	.16520-001	29.833	-.10278+001	-.16403+003
19	.43981-003	125.540	-.12833+002	-.14635+003
20	.33937-001	33.518	-.70491+000	-.70479+002
21	.72450-001	33.648	-.84397+000	-.81545+002
22	.34335-001	33.810	-.24970+001	.91324+002
23	.17263-005	-60.452	-.33251+002	-.64318+001
24	.27559-003	-57.137	-.16233+002	-.11272+003
25	.26471-002	-114.743	-.90460+001	.16087+003

Table 8. Fourier coefficients for the pattern of the configuration in figure 4.

---FOURIER COEFFICIENTS FOR REAL(E)

FREQUENCY = .2000+008 HZ

WAVELENGTH = 14.9900 METERS

START = .000 DEGREES

NO. OF SAMPLES = 100

NO. OF TERMS = 10

THE PHASE CENTER IS LOCATED AT (.0000, .0000) METERS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	.0000	-.89494+001		.11486+002	141.182
1	7.4950	.14094+002	-.16630+002	.21799+002	49.718
2	14.9900	.33733+001	-.69183+001	.76969+001	64.006
3	22.4850	-.22592+001	-.58408+000	.23334+001	165.504
4	29.9800	.93269+000	.54030+000	.10779+001	-30.083
5	37.4750	-.51685+000	-.48985+000	.71210+000	136.537
6	44.9700	.30435+000	.44355+000	.53795+000	-55.545
7	52.4650	-.18713+000	-.39621+000	.43818+000	115.282
8	59.9600	.84189-001	.40718+000	.41579+000	-78.318
9	67.4550	-.59178-001	-.21361+000	.22165+000	105.485
10	74.9500	.52692-001	.53798+000	.54055+000	-84.406

****

START = 165.000 DEGREES

THE PHASE CENTER IS LOCATED AT (9.1440, 3.0480) METERS

THE PHASE CENTER HAS BEEN SHIFTED BY 8.6900 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-8.6900	-.63075+000		.11365+001	-123.710
1	-1.1950	.23433+002	.80336+001	.24771+002	-18.924
2	6.3000	.10126+002	-.38164+001	.10821+002	20.651
3	13.7950	.92319+000	-.19035+001	.21156+001	64.127
4	21.2900	-.32537+000	.84215+000	.90281+000	-111.124
5	28.7850	.27257+000	-.56454+000	.62690+000	64.228
6	36.2800	-.26813+000	.42945+000	.50628+000	-121.979
7	43.7750	.27192+000	-.34593+000	.44001+000	51.830
8	51.2700	-.26150+000	.30320+000	.40039+000	-130.776
9	58.7650	.30746+000	-.22843+000	.38303+000	36.611
10	66.2600	-.23220+000	.25362+000	.34386+000	-132.474

Larger angles often cause abnormal program termination. Another complication is that for the problem of figure 4, the sources are not all on a single axis since the box itself also radiates. Misalignment of multiple sources can be thought of as noise in the pattern data and the Prony process is known to be very susceptible to noise.

The assumed array axis is determined by the initial angle in the pattern data. If the pattern data is in the normal order, the initial angle is zero degrees and the assumed array axis lies in this direction, ie, along the X axis. If the pattern data were re-ordered so that the initial angle is 90 degrees, for example, the assumed array axis would coincide with that angle - the Y axis. Thus in ADPAP, the assumed array axis may be changed by selecting the starting angle of the pattern to be processed. The angle specified must coincide with an angle in the pattern data; eg, if the pattern is specified every two degrees starting at zero, only even angles may be used to orient the assumed array axis.

Making use of the axis orientation option as well as the shifting theorem option of ADPAP, the Prony residues and poles listed in table 7 may be obtained. The array axis was rotated 165 degrees to align the assumed array axis to the nearest whole angle of the true array axis shown in figure 5 (the pattern data was calculated by NEC in 5° increments). The phase center has also been shifted to coincide with the location of the 35-foot whip at the box center.

Only the first three poles in table 7 have significant residues with the number 1 and 3 poles close to the imaginary axis. Thus two sources are identified: one at the center of the assumed array axis and the other at S = +8.4185 meters. Further iterations of Prony with more poles lead to the same results. Noise introduced by misalignment of the array axis and radiation from parts of the box are responsible for the large real parts of the poles. Most of the errors in pole location and source strength are attributed to the same cause. When the problem is over specified by requiring too many poles, additional curve fitting sources are introduced which are not clearly different from the true source. Thus a possible source is indicated by pole number 2 at S = -8.4258 meters.

Table 8 lists Fourier coefficients for the pattern of the configuration in figure 4. The first set of coefficients is for direct application of the Fourier process to the pattern data as it was originally generated (the phase center is at the origin in figure 4). Multiple sources are indicated between the origin and 29.98 meters along the X axis. Iteration of the shifting theorem does not clearly identify the sources when the assumed array axis is badly misaligned. The second set of coefficients in table 8 results from reorienting the assumed array axis to 165° and shifting the phase center to the center of the box, as was the case for Prony. After incrementally shifting along the array axis, the maximum amplitudes were found with a shift of 8.69 meters. The two sources are indicated by coefficients 1 and 2 at a half-wavelength separation. The strongest source is at the array center, which is the location of the driven antenna. Again there is good correlation between Fourier and Prony for the source amplitude and phase.

3.3 ANTENNAS ON A SHIP

Figure 6 shows a sketch of a wire grid model of the PGG, a small ship about 58 meters long. The two 35-foot whips are about 23.24 meters apart. Figure 7 shows a top view of the PGG giving exact antenna locations.

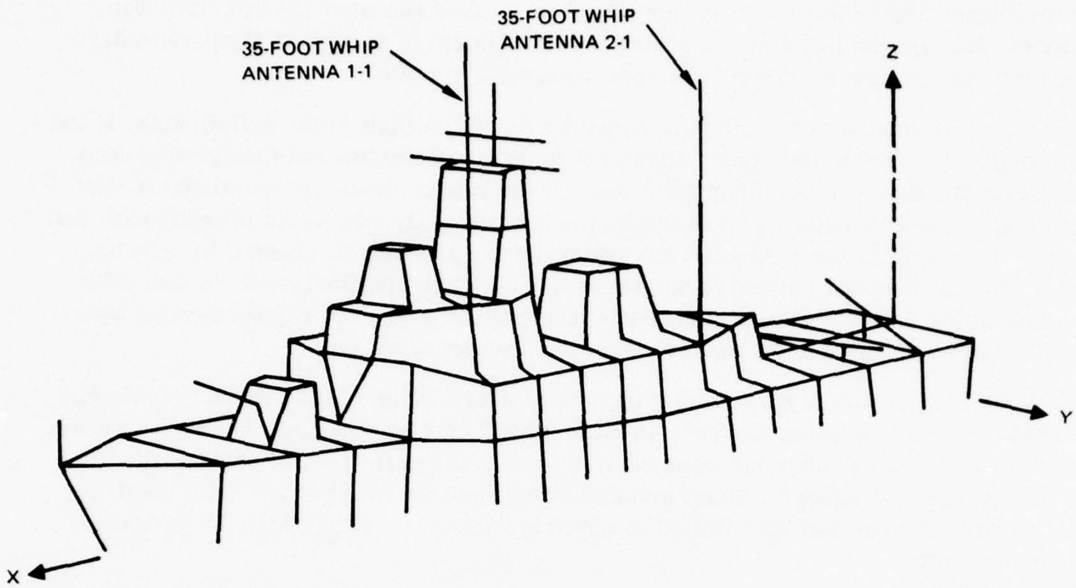

Figure 6. Sketch of wire grid model of the PGG. Antenna 2-1 is driven at 10 MHz.

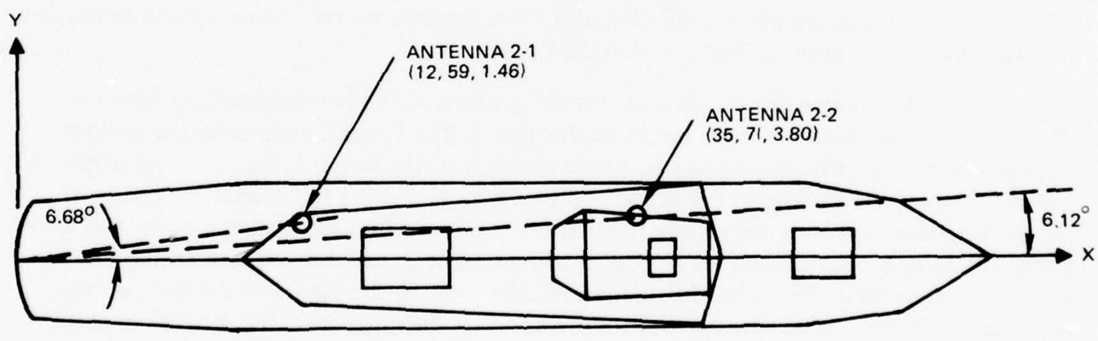

Figure 7. Top view of the PGG showing antenna locations. Antenna coordinates are in meters.

NEC was used to model the 35-foot whips and the ship, using a wire grid scheme for conducting surfaces as shown in figure 6. Patterns were calculated using the origin shown as the phase center. Azimuth patterns were determined in 5-degree increments for a 5-degree elevation angle. The pattern of the aft-most whip (antenna 2–1) at 10 MHz was processed using ADPAP. At 10 MHz, the antennas are sufficiently separated for resolution by both Prony and Fourier processing. Some of these results are shown in tables 9 and 10.

Table 9 lists the Prony residues and poles for the 10-MHz pattern of antenna 2–1. The assumed array axis is set at 5 degrees to coincide, within the nearest whole degree of pattern data, to the actual alignment of the whips (see figure 7). The phase center was not moved from the origin defined in figures 6 and 7. A number of iterations of the Prony process for 10 through 50 poles were performed. The results of table 9 are typical. Two poles (pole numbers 3 and 4) have relatively large residues and are located at 13.3 and 36.5 meters along the assumed array axis. The whip separation is correctly predicted. A small error in specific location is seen. Also, the poles are not as close to the imaginary axis in the complex spatial domain as they were for the simpler arrays. Both discrepancies are attributable to noise introduced by (1) misalignment of the assumed array axis with the true sources and (2) radiation from structural members of the ship.

Table 10 lists the Fourier coefficients derived from the pattern of antenna 2–1. The assumed array axis is set at 5 degrees, as in the case of Prony processing. The shifting option is applied in small increments to shift the phase center along the array axis in search of peaks in the amplitudes. Three of the iterations are shown to illustrate this approach. One peak occurs at 11.99 meters (coefficient number 1 in set 2) and another at 23.994 meters (coefficient number 2 in set 3). The locations (S) are with respect to the original phase center. If smaller increments in the shifting theorem are used, a more exact determination of the source locations is possible. This process is tedious and may require an excessive number of iterations to equal the accuracy of Prony processing.

3.4 SUMMARY

The preceding sections have described how Prony processing may be applied to antenna patterns to determine source locations. Section 3.3 demonstrated that Prony processing can accurately identify the principal radiators even for the cluttered shipboard environment. This technique could be a very useful tool during the shipboard antenna arrangement design in cases where one or more parasitic structures are present. Not only can these structures be located by the Prony poles, but their relative importance can be deduced from the size of the residues.

Prony processing is not the only possible approach, as demonstrated via Fourier series processing. But the chief advantages of Prony are that (1) unlike the Fourier series approach, Prony processing can resolve sources separated by less than one-half wavelength, and (2) the solution is found with fewer iterations. Perhaps with experience or some suitable eigenvalue technique (suggested by Lawrence Livermore Laboratory)[2], only one or two iterations may be required. The only disadvantage of Prony is its susceptibility to noise in the pattern data. This problem is now receiving considerable attention from the technical community.

Table 9. Prony residues and poles for the pattern of the configuration in figure 6.

```
---COMPLEX PRONY

     FREQUENCY =          .1000+008     HZ

     WAVELENGTH =            29.9800      METERS

     START =      5.000 DEGREES

     NO. OF SAMPLES =    30

     NO. OF POLES =    15

THE PHASE CENTER IS LOCATED AT (       .0000,       .0000)    METERS
```

	RESIDUES MAG	RESIDUES PHASE	POLES REAL	POLES IMAG
1	.99590-001	-154.046	-.37893+001	-.31746+001
2	.41057-001	-136.744	.83704+001	-.57037+001
3	.97277+000	15.985	-.48387+001	.13344+002
4	.17631+000	-137.216	.11267+001	.36545+002
5	.45120-007	-28.783	.75569+002	.12252+002
6	.88001-002	-94.442	.32966+001	.68259+002
7	.66177-003	-127.838	-.58016+001	-.90979+002
8	.10252-002	38.467	-.64452+001	.15480+003
9	.79269-004	-141.400	.11263+002	.21049+003
10	.75719-004	-13.846	-.28217+002	-.53287+002
11	.42668-003	-140.575	-.55393+001	-.12272+003
12	.11883-002	31.778	-.73436+001	.12480+003
13	.22134-003	36.503	-.10241+002	-.17588+003
14	.44996-003	48.330	-.56876+001	-.20100+003
15	.71885-004	70.641	-.22807+002	.18725+003

Table 10. Fourier coefficients for the pattern of the configuration of figure 6.

---FOURIER COEFFICIENTS FOR REAL(E)

FREQUENCY = .1000+008 HZ

WAVELENGTH = 29.9800 METERS

START = 5.000 DEGREES

NO. OF SAMPLES = 100

NO. OF TERMS = 10

THE PHASE CENTER IS LOCATED AT (.0000, .0000) METERS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	.0000	-.30765-001		.46274+000	-93.812
1	14.9900	.10464+001	-.31315+000	.10922+001	16.661
2	29.9800	-.34211+000	-.17358+000	.38363+000	153.098
3	44.9700	.42590-001	.23682+000	.24062+000	-79.805
4	59.9600	-.56157-001	-.15455+000	.16444+000	109.969
5	74.9500	.49434-001	.12932+000	.13845+000	-69.081
6	89.9400	-.45752-001	-.10365+000	.11330+000	113.818
7	104.9300	.42355-001	.86334-001	.96164-001	-63.867
8	119.9200	-.39845-001	-.73932-001	.83986-001	118.322
9	134.9100	.38049-001	.64499-001	.74886-001	-59.463
10	149.9000	-.36697-001	-.57136-001	.67906-001	122.712

****

THE PHASE CENTER HAS BEEN SHIFTED BY 2.9980 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-2.9980	-.31674+000		.51018+000	-128.378
1	11.9920	.12864+001	-.64409-001	.12880+001	2.866
2	26.9820	-.24370+000	-.42811+000	.49261+000	119.650
3	41.9720	-.19937-001	.36623+000	.36677+000	-93.116
4	56.9620	-.42227-001	-.22204+000	.22602+000	100.768
5	71.9520	.44513-001	.17641+000	.18194+000	-75.838
6	86.9420	-.44877-001	-.13782+000	.14494+000	108.037
7	101.9320	.43462-001	.11365+000	.12168+000	-69.073
8	116.9220	-.42261-001	-.96858-001	.10568+000	113.572
9	131.9120	.41368-001	.84329-001	.93929-001	-63.870
10	146.9020	-.40643-001	-.74634-001	.84983-001	118.571

****

THE PHASE CENTER HAS BEEN SHIFTED BY 5.9960 METERS ALONG ARRAY AXIS

N	S(METERS)	I(N)	J(N)	AMP(N)	PHASE
0	-5.9960	-.45619+000		.52492+000	-150.351
1	8.9940	.12642+001	.15430+000	.12736+001	-6.958
2	23.9840	.11377-001	-.60041+000	.60052+000	88.914
3	38.9740	-.14114+000	.39577+000	.42018+000	-109.627
4	53.9640	.42724-002	-.20830+000	.20835+000	88.825
5	68.9540	.11987-001	.16005+000	.16049+000	-85.717
6	83.9440	-.19792-001	-.12030+000	.12192+000	99.343
7	98.9340	.22657-001	.97915-001	.10050+000	-76.971
8	113.9240	-.24408-001	-.82954-001	.86470-001	106.396
9	128.9140	.25599-001	.72024-001	.76438-001	-70.433
10	143.9040	-.26373-001	-.63668-001	.68914-001	112.501

4.0 APPLICATION TO PATTERN DATA COMPRESSION

Prony processing of pattern data to obtain source location and strength information requires both magnitude and phase of the pattern. The pattern data used in the examples of section 3.0 consist of from 72 to 360 complex numbers, depending on whether data are supplied in 5- or 1-degree increments. For acceptable accuracy, anywhere from 20 to 40 poles were determined.

If, on the other hand, the source location and strength information is not required, but instead the objective is to reproduce the original pattern data from the Prony derived exponential series, fewer poles will be required. Either fewer poles need be determined in the first place or some of the poles of a set may be truncated from the series. The number of poles required for adequate representation is determined in part by the application and in part by trial. In fact, the original pattern data need not include the phase, if this information is not required by the application. Of course, the Prony residues and poles would lose their physical identities, but may still be used to retrieve the original pattern data.

Large volumes of pattern data are already measured on the NOSC pattern range as a routine part of shipboard antenna arrangement procedures. (These data do not as a rule include phase information.) The pattern data are used immediately at the time of collection for some statistical processing, some are plotted and all may be stored for future reference on magnetic tape. Representation of these data as sets of Prony derived complex numbers could significantly reduce storage requirements. (However, a Fourier series representation could be equally as good in this case.) Instead of retaining 360 bits of information per pattern, only a few pairs of complex numbers need be stored, depending on the required accuracy.

Another application of Prony for pattern storage and retrieval is in direction finding (DF) using adaptive arrays. Especially in the case where an adaptive array uses a number of drastically dissimilar antennas, the Prony representation of antenna patterns could be useful.

An adaptive array automatically adjusts the phase and amplitude of each receive channel to optimize the composite signal to some predetermined criterion, usually signal-to-noise ratio. The array processor makes no use of antenna locations, antenna patterns, and phase delays in coaxial lines and multicoupler filters. Hence, the processor generally does not have enough information to determine the array pattern. For purposes of adaptive nulling out of interfering signals, the exact antenna pattern shape is unnecessary information, but it is essential for a DF capability.

The information available from the array processor after it has adapted is a set of complex weights (amplitude and phase) for each signal channel. This information could be combined with the phase delay in each channel path plus accurate antenna patterns, to be derived from Prony poles and residues, and interpreted for DF information. Based on a recent paper by Dr. Kazuki Takao and Dr. S. Keiichi,[10] it may also be possible to distinguish between ground and skywave signals or other multipath signals using Prony processing.

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10. Faculty of Engineering, Kyoto University, Kyoto, Japan, EMCJ 77-34, Measurement of TV Ghost Waves by a Small Antenna, by K. Takao and S. Keiichi, 28 October 1977.

Another useful application of Prony processing is to couple two numerical antenna modeling techniques, method of moments and the geometric theory of diffraction (GTD). A general purpose electromagnetics code for communications antennas which employs GTD is under development at Ohio State University (OSU).[11] This code computes the fields of one or more dipoles including the effects of nearby scattering objects composed of flat plates and cylinders. The code is valid at frequencies where asymptotic techniques are appropriate. NEC, which has previously been described, is a general purpose antenna code which solves the boundary value problem for the antenna currents using the method of moments. The cost of solving for the currents on an antenna and all appropriate structures using NEC soon becomes unreasonable as the frequency is increased. The solution is to switch to an asymtotic technique like the GTD code of OSU. However, for some antennas, a simple dipole representation and an equivalent dipole array may not be readily obvious. Using NEC, the antenna can be accurately modeled by itself; the current distribution is found and the antenna pattern calculated (in free space). Prony processing of the pattern will then replace the antenna currents with an array of dipoles whose strengths (both magnitude and phase) and locations are specified by the residues and poles. These sources are then used in the GTD code to calculate the patterns, including the scattering from nearby structures. This technique has shown a considerable savings in computer costs for two test cases (not shown here) and warrants further study.

5.0 CONCLUSIONS

The technical objective of this study is to assess the feasibility and the utility of inverting the far field patterns of shipboard antennas. The feasibility of using Prony's method is illustrated in sections 2.0 and 3.0. Section 2.0 describes the Prony method for deriving the coefficients and arguments of an exponential series representation of far field antenna patterns. These coefficients and arguments can be interpreted to reveal source strength and location as demonstrated in section 3.0. The question of utility of the Prony transformation or inversion is not as easily answered as is the question of feasibility. Two applications were considered; data characterization and data compression.

An application of Prony's method for data characterization is the imaging of radiation sources. Section 3.0 demonstrates that the Prony derived coefficients and arguments can provide source strength and location information for a given shipboard antenna pattern. This is possible because of the exponential relation between an array of sources and their pattern. The usefulness of this approach for identifying shipboard sources from their patterns is limited by two disadvantages of Prony's method. The principal disadvantage of Prony's method for data characterization is its susceptibility to noise in the pattern data. Noise introduces errors in the source representation. The noise in the pattern data is introduced by the misalignment of sources and/or the distributed nature of radiation from deliberate antennas with parasites. The second disadvantage is that the interpretation of the Prony coefficients and arguments is not always straightforward. Considerable experience with the application of Prony processing and some knowledge of the primary source locations are prerequisites. Even then the process of interpretation of the Prony data is not always explicit. As a result, the cost of the engineering time far outweighs the cost of the computational time.

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11. Ohio State University Electroscience Laboratory, TR 4508-8 (784 508), User's Manual for Plates and Cylinder Computer Code, by RJ Marhefka, March 1978.

Of course, Prony's method is not the sole technique for successfully extracting source information from pattern data. One other approach, also demonstrated in section 3.0, is the use of a truncated Fourier series. The accuracy of this approach is not as sensitive to noisy data and there is no difference in computation costs. The principal disadvantage of the Fourier series approach is that resolution of closely spaced sources is, at best, one-half wavelength. However, in one example of section 3.0, Prony's method was able to resolve two sources separated by only one-third wavelength.

Another application of Prony's method for data characterization is that of developing a model for a radiating platform that, in a perturbation sense, could be used to predict the new behavior when the geometry is modified. For example, the data from the NOSC scale model range could be used to design and evaluate antenna systems for ships, especially when it is not practical to model all possible configurations. However, the noise problem and interpretation problem severely limit the use of Prony's method in shipboard antenna design. Further development will be necessary before Prony's method becomes a practical tool for the shipboard antenna designer. Since further development of Prony's method is being pursued under other applications, it is appropriate to await possible improvements from these efforts rather than to continue in this one.

Prony's method can also be applied to data compression. When the objective is data storage and retrieval of large volumes of data, Prony derived residues and poles provide an abbreviated storage mode. However, no physical significance is attached to the Prony derived coefficients and arguments. They are used only for reconstruction of the original data. As an example, a 180-point representation of a pattern could be represented by 6 to 10 data points.

The data compression attributes of Prony's method can also be used to develop simpler equivalent source distributions for complex antennas. Currently, this equivalent sources concept is being pursued within the method of moments and geometric theory of diffraction efforts at NOSC.

6.0 REFERENCES

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10. Faculty of Engineering, Kyoto University, Kyoto, Japan, EMCJ 77–34, Measurement of TV Ghost Waves by a Small Antenna, by K. Takao and S Keiichi, 28 October 1977.

11. Ohio State University Electroscience Laboratory, TR 4508–8 (784 508), User's Manual for Plates and Cylinder Computer Code, by RJ Marhefka, March 1978.

APPENDIX A

ANGLE DOMAIN PATTERN
ANALYSIS PROGRAM
(ADPAP)

USER'S GUIDE

A.1 INTRODUCTION

Angle Domain Pattern Analysis Program (ADPAP) performs either a Prony or a Fourier series analysis on radiation patterns for data supplied in equal degree increments for any π interval in the range $0 \leqslant \theta \leqslant 2\pi$. Both magnitude and phase of the radiation pattern are required for correct results. The Fourier series expansion may be applied to either the real or the imaginary part of the pattern.

The shifting theorem may be applied to both the Fourier and the Prony analysis to move the apparent phase center of the radiation pattern. One shift in both X and Y directions may be applied before selection of the π interval for processing. Multiple shifts along the assumed array axis may be applied after selection of the π interval and prior to the initiation of the Fourier or Prony analysis. The shift can be specified in feet or in metres.

A.2 DATA INPUT FORMAT

ADPAP uses a free format input – all data items on a single line are separated by commas.

Free format input is accomplished by suspending the standard storage control table element which normally checks for record type when reading from a file. For the NOSC UNIVAC 1110 machine, this is accomplished at time of compilation by including the following commands in the run stream between the compilation and map:

```
@COPY ,S N*FTNLIB.
@ASM ,LI F2SCT ,F2SCT
    F$SCT 33 ,111 ,0
    END
```

A.3 DATA CARDS

ADPAP uses a data input format similar to NEC – Method of Moments[1]. Each line of data begins with a two-letter mnemonic which determines how the rest of the data on that line will be interpreted by the code. Each data set must begin with at least one comment card. Any number of comment cards may be used, but the last one must be a CE comment card. (If only one line of comments is required, then that card must be a CE card.) The order of the other cards is unimportant except that the XQ causes execution of the program for the parameters set on all previous cards back to the last XQ card, if there is one. Use of the NX card re-initializes all program parameters, allowing introduction of a new problem within the same data stream.

All data cards except for the comment cards have a two letter mnemonic followed by two integer fields and two floating fields. One other special case also differs from this format which is explained under the Pattern Data Command (PD) card.

1. Naval Ocean Systems Center Technical Document 116, Numerical Electromagnetic Code (NEC) – Method of Moments, by GJ Burke and AJ Poggio, 18 July 1977.

COMMENT CARDS (CM, CE)

CM — COMMENTS —

. .

. .

. .

CE — COMMENTS —

NOTES

- Any number of CM cards may be used.
- The last comment must be a CE card.
- For only one line of comments, use a single CE card.
- A maximum of one CE card may be used.
- ALL comment cards must appear first in the data set.
- At least one comment card must follow an NX card.

END of RUN (EN)

PURPOSE: To indicate to the program the end of all execution.

CARD: EN

FOURIER ANALYSIS (FA)

PURPOSE: To initiate a Fourier analysis

CARD: FA, NTERMS, NSAMPS, START

PARAMETERS:

Integers

NTERMS – Desired number of Fourier coefficients

NSAMPS – Number of numerical integration increments used for Euler Equations

Decimal Numbers

START – Initial angle of 180° of patterns used in analysis in degrees.

NOTES:

. The range of source locations is (NTERMS) meters.
The possible source locations are at $S = \pm n(\lambda/2)$, n = 0, 1, 2, ..., NTERMS.

. NTERMS greater than 0; REAL $(E(\theta))$ is processed
NTERMS less than 0; IMAG $(E(\theta))$ is processed

. Only 180° of pattern may be analyzed at a time. An 180° sector of the pattern may be used beginning with the angle START (in degrees). Symmetry of pattern is always assumed.

NEW PROBLEM (NX)

PURPOSE: To signal the end of one problem and the beginning of an entirely new problem

CARD: NX

NOTES:

. This card will re-initialize all control parameters and arrays.

. The card that directly follows the NX card must be a comment card.

PRONY ANALYSIS (PA)

PURPOSE: To initiate a Prony analysis

CARD: PA, NPOLES, 0, START

PARAMETERS:

Integers

NPOLES – Desired number of poles

0 – Blank

Decimal Numbers

START – Initial angle of 180° of pattern used in analysis in degrees

NOTES:

. The number of pattern data samples is equal to twice the number of poles

. Only 180° of pattern may be analyzed at a time. Any 180° sector of the pattern may be used beginning with the angle START (in degrees). Symmetry of pattern is always assumed.

PATTERN DATA (PD)

PURPOSE: To specify the sampling parameters and data of the radiation pattern

CARD: (Format A)

PD, NF, ICVR, F, ELW

(Format B)

0 (1)	E (1)	Phase (1)
.	.	.
.	.	.
.	.	.
0 (NF)	E (NF)	Phase (NF)

PARAMETERS:

Integers

NF – Number of field points
ICVR – Special processing

IF = −1, Solves for amplitude of an elementary dipole
IF = 0, point sources
IF = +1, Solves for amplitude of $\lambda/2$ dipoles

Decimal Numbers

F – Frequency in Hz
ELW – If ICVR = −1, *effective* length in wavelengths of elementary dipole
(I) – Pattern angle of sample I
E (I) – Magnitude of field of sample I
Phase (I) – Phase of field of sample I

NOTES:

. The first card follows format A with the definitions given above. The next NF cards have format B giving the azimuth pattern angle, field strength magnitude and phase as indicated.

PRINT CONTROL (PT)

PURPOSE: To control printing of pattern data

CARD: PT, IPT1, IPT2

PARAMETERS:

Integers

IPT1 – = 0, no output of original pattern data
= 1, prints original pattern data in equal cosine steps

IPT2 – = 0, no output of generated pattern
= N, N greater than 0, computes and prints pattern using Fourier coefficients for $\theta = 0, 180°$ in N equal steps

Decimal Parameters

No parameters

NOTES:

. To erase a previous PT specification, either use a new PT spec or use PT, 0, or use NX to re-initialize all arrays.

. If no PT is specified in a given run, no pattern data will be output.

SHIFTING THEOREM (SF)

PURPOSE: To multiply the pattern data by a phase shift

CARD: SF, ITER, UNITS, X, Y

PARAMETERS:

Integers

ITER – less than 0; resets pattern to original pattern
= 0; shifts phase center to location (X, Y) greater than 0; number of iterations of shifting theorem applied along array axis

Decimal Numbers

X – ITER = 0; X = shift along X axis
ITER *greater than* 0; X = initial shift along array axis

Y – ITER = 0; Y = shift along Y axis
ITER greater than 0; Y = increment of shift along array axis

NOTES:

. For ITER greater than 0, the factor exp (ikXcos θ) is applied to the pattern after incrementation in equal cosine steps and after rotation of the pattern by START (specified by the FA and PA commands).

. For ITER = 0, the factor exp (−ik(Xcos + Ysin)) is applied to the initial pattern.

. For ITER less than 0, all previous SF specs are erased and the original pattern will be processed.

. The array axis is the positive X axis or the line defined by the angle START on the FA and PA cards.

. TWO SF cards may be used consecutively before the XQ so that both shifting options can be used.

EXECUTE (XQ)

PURPOSE: To cause program execution at desired points in the data stream.

CARD: XQ

NOTES:

. The XQ card is required to cause execution of the program.

APPENDIX B

ANGLE DOMAIN PATTERN ANALYSIS PROGRAM (ADPAP)

PROGRAM LISTING

```
C*****************************************************************************
C************** ANGLE DOMAIN PATTERN ANALYSIS PROGRAM*****************
C*****************************************************************************
C
C---    FOURIER SERIES EXPANSION   ---
C
C       DEVELOPED BY J. ROCKWAY AND J. C. LOGAN
C       APRIL 1978
C
C---   ANGLE DOMAIN PRONY  ---
C
C      ADAPTED FROM LLL CODE BY J. C. LOGAN
C      APRIL 1978
C
      COMPLEX E(100),EP(360),A(100),ALPHA(100),EPP(360),EO(100),Z,SUM
      DIMENSION CI(51),CJ(50),S(51),AMP(100),PH(100)
      DIMENSION AC(100),AD(100),AP(360),SCALE(2),EX(100),EY(100)
      CHARACTER BLANK/' '/,SLASH/'/'/,CARD(78)
      CHARACTER*2 IR(36),IT(10)
      DATA IT/2HCM,2HCE,2HPD,2HPA,2HXQ,2HNX,2HEN,2HFA,2HPT,2HSF/
      DATA PI/3.1415927/
      DATA SCALE/1.,0.304801/
      DEFINE FILE 9(SDF)
C---INITIALIZE DATA
  500 IPRONY=0
      IFOUR=0
      IREAL=1
      START=0.
      SX=0.0
      SY=0.0
      SA=0.
      DSA=0.
      EL=0.
      ELW=0.
      ICUR=0
      IPT1=0
      IPT2=0
      ITER=0
      DO 501 I=1,360
      AP(I)=0.
      EP(I)=CMPLX(0.,0.)
      EPP(I)=EP(I)
  501 CONTINUE
      DO 502 I=1,100
      E(I)=CMPLX(0.,0.)
      ALPHA(I)=CMPLX(0.,0.)
      A(I)=ALPHA(I)
      AD(I)=0.
      AC(I)=0.
  502 CONTINUE
      WRITE(6,100)
  100 FORMAT(1H1,T18,'***** ANGLE DOMAIN PATTERN ANALYSIS PROGRAM '
     1'*****'///T36,'COMMENTS'/T36,'--------'//)
    1 READ(5,101) IR
  101 FORMAT(36A2)
      WRITE(6,102) (IR(I),I=2,36)
```

```
 102  FORMAT(5X,35A2)
      IF(IR(1).EQ.IT(1)) GO TO 1
      IF(IR(1).EQ.IT(2)) GO TO 200
      WRITE(6,103)
 103  FORMAT(2X,'READ ERROR IN INPUT DATA - STOP!')
      STOP
 200  ITEMP1=0
      ITEMP2=0
      TEMP1=0.
      TEMP2=0.
      READ(5,104) IR(1),CARD
 104  FORMAT(A2,78A1)
      CARD(1)=BLANK
   5  IF(CARD(78).EQ.BLANK) GO TO 6
      WRITE(9,105) CARD
 105  FORMAT(78A1)
      READ(5,105) CARD
      GO TO 5
   6  DO 7 I=2,78
      IF(CARD(I).EQ.BLANK) CARD(I)=SLASH
      IF(CARD(I).EQ.SLASH) GO TO 8
   7  CONTINUE
   8  CONTINUE
      WRITE(9,105) CARD
      END FILE 9
      REWIND 9
      READ(9,*,ERR=1000) ITEMP1,ITEMP2,TEMP1,TEMP2
      REWIND 9
C
C---  COMMANDS:
C
C   CM   COMMENTS   1
C   CE   COMMENTS   2
C   PD   PATTERN DATA   3
C   PA   PRONY ANALYSIS   4
C   XQ   EXECUTE   5
C   NX   NEW PROBLEM   6
C   EN   END EXECUTION   7
C   FA   FOURIER ANALYSIS   8
C   PT   PRINT PATTERNS   9
C   SF   SHIFTING THEOREM   10
C
      WRITE(6,106) IR(1),ITEMP1,ITEMP2,TEMP1,TEMP2
 106  FORMAT(//2X,3H***,A2,2I4,2E15.5)
      IF(IR(1).EQ.IT(3)) GO TO 30
      IF(IR(1).EQ.IT(4)) GO TO 40
      IF(IR(1).EQ.IT(5)) GO TO 50
      IF(IR(1).EQ.IT(8)) GO TO 800
      IF(IR(1).EQ.IT(9)) GO TO 900
      IF(IR(1).EQ.IT(6)) GO TO 500
      IF(IR(1).EQ.IT(10)) GO TO 600
      IF(IR(1).EQ.IT(7)) GO TO 700
      WRITE(6,115)
 115  FORMAT(///2X,'COMMAND IS UNDEFINED - STOP!')
      STOP
C
C---  READ IN PATTERN DATA
```

```
C
   30 NF=ITEMP1
      ICUR=ITEMP2
      F=TEMP1
      ELW=TEMP2
      IF(F .EQ. 0.) F=299.8E+06
      W=2.998E+08/F
      PK=6.2831853/W
      DO 31 I=1,NF
      READ(5,*) AP(I),X,Y
      YRAD=0.0174533*Y
      XR=X*COS(YRAD)
      YI=X*SIN(YRAD)
   31 EO(I)=CMPLX(XR,YI)
      IF(ITEMP2) 34,200,32
   32 IF(ELW .EQ. 0.) ELW=0.5
      EL=ELW*W
      DO 33 I=1,NF
      APR=AP(I)*0.0174533
      FTR=-SIN(APR)/(COS(0.5*PK*EL*COS(APR))-COS(0.5*PK*EL))
   33 EO(I)=EO(I)*CMPLX(0.,FTR/60.)
      GO TO  200
   34 IF(ELW .EQ. 0.) ELW=0.125
      EL=ELW*W
      DO 35 I=1,NF
      APR=AP(I)*0.0174533
      FTR=-30.*PK*EL*SIN(APR)
   35 EO(I)=EO(I)*CMPLX(0.,1/FTR)
      GO TO 200
C
C---  SET PARAMETERS FOR PRONY ANALYSIS
C
   40 NPOLES=ITEMP1
      START=TEMP1
      NSAMPS=2*NPOLES
      IPRONY=1
      IFOUR=0
      GO TO 200
C
C---   SET UP PARAMETERS FOR SHIFTING THEOREM
C
  600 IF(ITEMP1) 630,620,610
  610 ITER=ITEMP1
      DSA=TEMP2*SCALE(ITEMP2+1)
      SA=TEMP1*SCALE(ITEMP2+1)
      GO TO 200
  620 SX=TEMP1*SCALE(ITEMP2+1)
      SY=TEMP2*SCALE(ITEMP2+1)
      GO TO 200
  630 SA=0.
      ITER=0
      DSA=0.
      SX=0.0
      SY=0.0
      GO TO 200
C
C---  SET UP PARAMETERS FOR FOURIER SERIES
```

```
C
 800    IFOUR=1
        IPRONY=0
        NTERMS=ABS(ITEMP1)
        NSAMPS=ITEMP2
        START=TEMP1
        IF(ITEMP1 .LT. 0) IREAL=0
        GO TO 200
C
C---     PRINT CONTROL
C
 900    IPT1=ITEMP1
        IPT2=ITEMP2
        GO TO 200
C
C---    PROGRAM EXECUTION
C
  50    CONTINUE
        IF(SX .EQ. 0.0 .AND. SY .EQ. 0.0) GO TO 693
C
C---    APPLY SHIFTING THEOREM
C
        DO 691 I=1,NF
        APR=AP(I)*0.0174533
        ARG=-PK*(SX*COS(APR)+SY*SIN(APR))
        Z=CMPLX(0.,ARG)
        EPP(I)=EO(I)*CEXP(Z)
691     CONTINUE
        GO TO 692
693     CONTINUE
        DO 694 I=1,NF
 694    EPP(I)=EO(I)
 692    CONTINUE
C---    ITER = # OF ITERATIONS OF SHIFTING THEORM
        DO 999 ISTEP=1,ITER
        WRITE (6,127)
 127    FORMAT(///T36,3('*****')//)
        WRITE(6,128) SX,SY
128     FORMAT(//5X,'THE PHASE CENTER IS LOCATED AT (',
       1F10.4,',',F10.4,')',3X,'METERS'//)
        WRITE(6,117) F,W
117     FORMAT(//5X,'FREQUENCY =',E15.4,
       1'    HZ'//5X,'WAVELENGTH =',F15.4,'     METERS'//)
C
C---SELECT 2PI RANGE TO BE PROCESSED
C
        IF(START .EQ. 0.) GO TO 10
        FIN=START+180.
        IF(FIN .GT. 360.) FIN=FIN-360.
        DO 11 I=1,NF
        IF(START .EQ. AP(I)) JS=I-1
        IF(FIN .EQ. AP(I)) JF=I-1
11      CONTINUE
        ISKIP=JF-JS
        IF(ISKIP) 12,13,14
12      CONTINUE
        LIM=NF-JS+1
```

```
      DO 15 I=1,LIM
      EP(I)=EPP(JS+I)
15    CONTINUE
      DO 16 I=1,JF+1
      EP(I+LIM)=EPP(I)
16    CONTINUE
      GO TO 19
14    CONTINUE
      DO 17 I=1,NF
      EP(I)=EPP(I+JS)
17    CONTINUE
      GO TO 19
13    WRITE(6,116)
116   FORMAT(2X,'ERROR IN START SPECIFICATION-STOP!')
      STOP
10    CONTINUE
      DO 18 I=1,NF
      EP(I)=EPP(I)
18    CONTINUE
19    CONTINUE
      DELCOS=-2./(NSAMPS-1)
C
C---  SET UP ANGLES FOR EQUAL COS SPACING
C
      AC(1)=1.
      AD(1)=57.2957795*ACOS(AC(1))
      DO 60 I=2,NSAMPS
      AC(I)=AC(I-1)+DELCOS
      AD(I)=57.2957795*ACOS(AC(I))
   60 CONTINUE
C
C---  DETERMINE PATTERN IN EQUAL COS
C     BY LINEAR INTERPOLATION
C
      E(1)=EP(1)
      E(NSAMPS)=EP(NF)
      DO 90 J=2,NSAMPS
      DO 70 I=2,NF
      JM=I
      IF(AP(I).GT.AD(J)) GO TO 80
   70 CONTINUE
   80 CONTINUE
      FCTR=(AP(JM)-AD(J))/(AP(JM)-AP(JM-1))
      E(J)=EP(JM)-(EP(JM)-EP(JM-1))*FCTR
   90 CONTINUE
      IF(ITER.EQ.0) GO TO 690
C
C---  APPLY SHIFTING THEOREM ALONG ARRAY AXIS
C
      DO 640 I=1,NSAMPS
      Z=CMPLX(0.,PK*AC(I)*SA)
      E(I)=E(I)*CEXP(Z)
640   CONTINUE
      WRITE(6,125) SA
125   FORMAT(///5X,'THE PHASE CENTER HAS BEEN SHIFTED BY',
     1F12.4,3X,'METERS ALONG ARRAY AXIS'//)
690   CONTINUE
```

```
      IF(IPT1 .EQ. 0) GO TO 901
C
C---  PRINT PATTERN IN EQUAL COS(THETA)
C
      DO 91 J=1,NSAMPS
      AMP(J)=CABS(E(J))
      PH(J)=180.*BTAN2(AIMAG(E(J)),REAL(E(J)))/PI
91    CONTINUE
      WRITE(6,107)
 107  FORMAT(//2X,'PATTERN DATA'//7X,'THETA',2X,
     1'COS(THETA)',10X,'REAL',10X,'IMAG',10X,'MAG',10X,'PHASE'/)
      WRITE(6,108) (AD(I),AC(I),E(I),AMP(I),PH(I),I=1,NSAMPS)
 108  FORMAT(5X,F7.3,F12.5,3E15.5,F12.3)
 901  CONTINUE
      WRITE(6,109) DELCOS
 109  FORMAT(//5X,'DELCOS =',E15.5)
      IF(IPRONY .EQ. 0) GO TO 801
C
C---  BEGIN PRONY PROCESS
C
      CALL PRONY(DELCOS,E,A,ALPHA,NPOLES,NSAMPS)
C---  CORRECT RESIDUES FOR SERIES NOT STARTING AT EXP(0)
C     NORMALIZE POLES TO UNITS OF DISTANCE
      DO 95 I=1,NPOLES
      A(I)=A(I)*CEXP(-ALPHA(I))
      Z=A(I)
      PH(I)=180.*BTAN2(AIMAG(Z),REAL(Z))/PI
      ALPHA(I)=ALPHA(I)/PK
      AMP(I)=CABS(A(I))
   95 CONTINUE
      WRITE(6,110)
 110  FORMAT(//2X,'---COMPLEX PRONY'//)
      WRITE(6,111) START,NSAMPS,NPOLES
 111  FORMAT(//5X,'START = ',F8.3,' DEGREES'//5X,'NO. OF SAMPLES =',
     1I5//5X,'NO. OF POLES =',I5//)
      IF(ICUR .EQ. -1) WRITE(6,130) ELW,EL
      IF(ICUR .EQ. 1) WRITE(6,129) ELW,EL
      WRITE(6,112)
 112  FORMAT(//16X,'RESIDUES',50X,'POLES'/12X,
     1'REAL',12X,'IMAG',12X,'MAG',10X,'PHASE',10X,'REAL',12X,'IMAG'/)
      WRITE(6,113) (I,A(I),AMP(I),PH(I),ALPHA(I),I=1,NPOLES)
 113  FORMAT(2X,I2,3E15.5,F15.3,2E15.5)
 801  IF(IFOUR .EQ. 0) GO TO 990
C
C---  BEGIN FOURIER SERIES ANALYSIS
C
C---  IF IREAL=0,EXPAND IMAG(E) IN FOURIER SERIES
C     IF IREAL=1,EXPAND REAL(E) IN FOURIER SERIES
      IF(IREAL .EQ. 0) GO TO 807
      DO 806 I=1,NSAMPS
      EX(I)=REAL(E(I))
806   EY(I)=AIMAG(E(I))
      GO TO 809
807   DO 808 I=1,NSAMPS
      EY(I)=REAL(E(I))
808   EX(I)=AIMAG(E(I))
809   CONTINUE
```

```
      CALL FOUR(-DELCOS,AC,EX,CI,CJ,NTERMS,NSAMPS)
C---  DETERMINE SOURCE STRENGTH AND PHASE
      IF(IREAL .EQ. 0) GO TO 810
      CALL FOURO(-DELCOS,EY,CO,NSAMPS)
      PH(1)=BTAN2(CO,CI(1))
      AMP(1)=CI(1)/COS(PH(1))
      PH(1)=57.2957795*PH(1)
      DO 805 I=1,NTERMS
      PH(I+1)=BTAN2(-CJ(I),CI(I+1))
      AMP(I+1)=CI(I+1)/COS(PH(I+1))
805   PH(I+1)=57.2957795*PH(I+1)
      WRITE(6,118)
118   FORMAT(//2X,'---FOURIER COEFFICIENTS FOR REAL(E)'//)
      GO TO 812
 810  CALL FOURO(-DELCOS,EY,CO,NSAMPS)
      PH(1)=BTAN2(CI(1),CO)
      AMP(1)=CI(1)/SIN(PH(1))
      PH(1)=57.2957795*PH(1)
      DO 811 I=1,NTERMS
      PH(I+1)=BTAN2(CI(I+1),CJ(I))
      AMP(I+1)=CI(I+1)/SIN(PH(I+1))
811   PH(I+1)=57.2957795*PH(I+1)
      WRITE(6,126)
126   FORMAT(//2X,'---FOURIER COEFFICIENTS FOR IMAG(E)'//)
812   CONTINUE
C---  DETERMINE SOURCE LOCATIONS
      DO 804 I=1,NTERMS+1
804   S(I)=0.5*(I-1)*W-SA
      WRITE(6,119) START,NSAMPS,NTERMS
 119  FORMAT(//5X,'START = ',F8.3,' DEGREES'//5X,
     1'NO. OF SAMPLES = ',I5//5X,'NO. OF TERMS = ',I5//)
      IF(ICUR .EQ. -1) WRITE(6,130) ELW,EL
 130  FORMAT(//5X,'CURRENTS OF ELEMENTARY RADIATORS :   EFFECTIVE LENG'
     1'TH =',F12.4,'  LAMBDA (OR',F12.4,'  METERS)')
      IF(ICUR .EQ. 1) WRITE(6,129) ELW,EL
  129 FORMAT(//5X,'DIPOLE CURRENTS :   TOTAL LENGTH = ',F12.4,
     1'  LAMBDA (OR',F12.4,'  METERS)')
      WRITE(6,120)
 120  FORMAT(//3X,'N',4X,'S(METERS)',6X,
     1'I(N)',12X,'J(N)',20X,'AMP(N)',8X,'PHASE'/)
      WRITE(6,121) S(1),CI(1),AMP(1),PH(1)
 121  FORMAT(3X,'0',F12.4,1E15.5,25X,E15.5,F12.3)
      WRITE(6,122) (I,S(I+1),CI(I+1),CJ(I),AMP(I+1),PH(I+1),I=1,NTERMS)
122   FORMAT(2X,I2,F12.4,2E15.5,10X,1E15.5,F12.3)
      IF(IPT2 .EQ. 0) GO TO 990
C---   CALCULATE AND WRITE  PATTERN FROM FOURIER COEFFICIENTS
      WRITE(6,123)
 123  FORMAT(//2X,'PATTERN FROM FOURIER SERIES'//7X,'THETA',10X,'REAL',
     110X,'IMAG',10X,'MAG',10X,'PHASE'/)
      XB=180./IPT2
      DO 802 I=1,IPT2+1
      BETA=(I-1)*XB
      BETAR=BETA*PI/180.
      SUM=CMPLX(0.,0.)
      DO 803 J=1,NTERMS+1
      ARG=PK*S(J)*COS(BETAR)+PH(J)*0.0174533
803   SUM=SUM+AMP(J)*CMPLX(COS(ARG),SIN(ARG))
```

```
      IF(BETAR .EQ. 0. .OR. BETAR .EQ. 180.) GO TO 711
      IF(ICUR) 720,730,710
  710 ARG=60.*(COS(0.5*PK*EL*COS(BETAR))-COS(0.5*PK*EL))/SIN(BETAR)
      SUM=SUM*CMPLX(0.,ARG)
      GO TO 730
  711 SUM=CMPLX(0.,0.)
      SUMA=0.
      PHASE=0.
      GO TO 740
  720 ARG=30.*PK*EL*SIN(BETAR)
      SUM=SUM*CMPLX(0.,ARG)
  730 CONTINUE
      SUMA=CABS(SUM)
      PHASE=180.*BTAN2(AIMAG(SUM),REAL(SUM))/PI
  740 WRITE(6,124) BETA,SUM,SUMA,PHASE
  124 FORMAT(5X,F7.3,3E15.5,F12.3)
  802 CONTINUE
  990 SA=SA+DSA
  999 CONTINUE
      GO TO 200
 1000 WRITE(6,114)
  114 FORMAT(2X,'READ ERROR ON UNIT 9 - STOP!')
  700 STOP
      END
```

```
      FUNCTION BTAN2(X1,X2)
C     BTAN2=0. WHEN X1=X2=0.
      IF(X1) 3,1,3
    1 IF(X2) 3,2,3
    2 BTAN2=0.
      RETURN
    3 BTAN2=ATAN2(X1,X2)
      RETURN
      END
```

```
      SUBROUTINE CROUT (C, X, Y, NCOLSC, NCOLSY)
C     FORTRAN              CROUT
       COMPLEX    SUM,HIGH,SUM1,DETERM
      COMPLEX A, C, X, Y
      DIMENSION A(50,51), C(50,50), X(50,1), Y(50,1), INDEX(50)

C
C     CROUT REDUCTION



C       THIS IS A CROUT REDUCTION PROGRAM WHICH CAN EITHER SOLVE FOR THE
C      SOLUTION OF A MATRIX EQUATION, OR AS A SPECIAL CASE, IT CAN COMPU
C      INVERSE OF A SPECIFIED N BY N MATRIX
C
C     THE TECHNIQUE IS CLEARLY EXPLAINED ON PAGES 429 TO 435 OF THE BOOK
C      F.B. HILDEBRAND, INTRODUCTION TO NUMERICAL ANALYSIS, PUBLISHED BY
C      MCGRAW HILL IN 1956

C       THIS PROGRAM SOLVES THE MATRIX EQUATION HX = B. THE H AND THE B
C      MATRICES ARE SPECIFIED PARAMETERS, AND THE PROGRAM CALCULATES TH
C      MATRIX.

C       H IS AN N BY N MATRIX OF NUMBERS, B IS AN N BY M MATRIX OF NUMB
C      AND X IS AN N BY M MATRIX OF NUMBERS TO BE DETERMINED. IF THE IN
C       OF THE H MATRIX IS DESIRED, ONE SETS THE B MATRIX EQUAL TO AN N
C      IDENTITY MATRIX, THUS X BECOMES THE INVERSE MATRIX.

C      THE A MATRIX SPECIFIED IN THE ARGUMENT OF THE SUBROUTINE IS AN
C      AUGMENTED MATRIX CONTAINING THE N BY N MATRIX H IN THE FIRST N BY
C     LOCATIONS, AND IS AUGEMENTED IN THE N + 1  TO N + M COLUMNS BY THE
C       MATRIX B.
C
C       NN IS BY DEFINITION EQUAL TO N + M.
C
C     THE SOLUTION MATRIX, X, IS STORED IN THE FIRST N BY M (ROWS BY COL

C      ELEMENTS OF THE AUGMENTED MATRIX A, THE ORIGINAL H AND B MATRICES
C     DESTROYED, AND SHOULD BE SAVED ELSEWHERE IF THEY ARE TO BE USED AF
C          USING THIS ROUTINE.

COMMENT----SOLVES CX=Y FOR X     C=COEFF MATRIX
COMMENT----COPIES C AND Y INTO A MATRIX
      N=NCOLSC
      M=NCOLSY
      NN=N+M
      DO 200 IQ1=1,N
      DO 200 IQ2=1,N
  200 A(IQ1,IQ2) = C(IQ1,IQ2)
      DO 210 IQ3=1,N
      DO 210 IQ4=1,M
  210 A(IQ3,IQ4+N) = Y(IQ3,IQ4)
       ABJC = 1.E-20
       JZ=N-1
       JA=N+1

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```
          DO 30 I = 1, N
   30     INDEX(I)=I


         DO 700 J=1,NN


         DO 800 II=1,N
         SUM=0.0
         I=INDEX(II)
         IF(II-J)33,34,34

C         THE SECTION AFTER STATEMENT 33 IS EQUIVALENT TO FORMULA 10.4.5 AN
C                 10.4.6 OF HILDEBRAND

    33    IF (II - 1) 9000,9200,9000
  9000    LLLL = II - 1

          DO 9100 K = 1,LLLL
          IPPP = INDEX(K)
  9100     SUM = SUM + A(I,K)*A(IPPP,J)

  9200     A(I,J) = (A(I,J)-SUM)/A(I,II)

           GO TO 800

C         THE SECTION AFTER STATEMENT 34 IS EQUIVALENT TO FORMULA 10.4.4
C               OF HILDEBRAND

    34    IF (J - 1) 8000,8200,8000
  8000     LLLL = J - 1

         DO 8100 K=1,LLLL
         IPPP=INDEX(K)
  8100   SUM=SUM+A(I,K)*A(IPPP,J)

  8200   A(I,J)=A(I,J)-SUM

   800     CONTINUE


         IF(J-N)41,700,700
      41 L=INDEX(J)

C              THIS SECTION SHIFTS AND REORDERS THE COLUMNS AND ROWS ( THEY
C          RESHUFFLED AT THE END OF THE PROCESS TO BE PUT IN THE ORIGINAL O

         KA=L
         HIGH=A(L,J)
         KZ=0

         DO 35 I=J,JZ
         JC=I+1
         L=INDEX(JC)
          IF (CABS(HIGH) - CABS(A(L,J))) 36,35,35
      36 HIGH=A(L,J)
```

```
          KA=L
          KZ=1
       35 CONTINUE
           IF (CABS(HIGH) - ABJC) 31,31,3200
  31      WRITE(3,32) ABJC
    32     FORMAT (2X,24HPIVOT ELEMENT LESS THAN ,E20.8)
     3200 DO 37 K=1,N
          KK=K
          IF(INDEX(K)-KA)37,38,37
       37 CONTINUE
       38 ITEMP=INDEX(J)
          INDEX(J)=INDEX(KK)
          INDEX(KK)=ITEMP
      700 CONTINUE
          IF(M)2000,1000,2000
     2000 L=N-1
           DO 39 J = JA,NN
           LL = 1
           DO 42 K = 1,N
  C        THIS SECTION IS USED TO SEE IF ONLY A SINGULAR TYPE SOLUTION IS P
           IF (CABS(A(K,J)) - 0.0) 43,42,43
       42 CONTINUE
          IZ=INDEX(N)
           IF (CABS(A(IZ,N)) - 1.E-2) 46,46,44
     44    CONTINUE
          WRITE(3,45)
  45      FORMAT(1X,'* ONLY SOLUTION IS ZERO VECTOR*')
          GO TO 10
     46    CONTINUE
          WRITE(3,45)
           GO TO 10
  C           THIS LOOP IS EQUIVALENT TO 10.4.7 OF HILDEBRAND
       43 DO 40 IJ=LL,L
          SUM1=0.0
          II=N-IJ
          I=INDEX(II)
          LLL=II+1
          DO 9300 K=LLL,N
          IP=INDEX(K)
     9300 SUM1=SUM1+A(I,K)*A(IP,J)
```

```
      A(I,J)=A(I,J)-SUM1
   40 CONTINUE
   39 CONTINUE
 1000    CONTINUE
C        THIS SECTION SHIFTS THE SOLUTION MATRIX INTO THE FIRST N BY M
C             LOCATIONS (ROWS BY COLUMNS)
      DO 400 I=1,N
       DO 400 J = JA,NN
      K=INDEX(I)
      L=J-N
  400 A(I,L)=A(K,J)
COMMENT----WRITE ANSWER INTO X MATRIX
      DO 250 IQ5 = 1,N
      DO 250 IQ6=1,M
 250  X(IQ5,IQ6) = A(IQ5,IQ6)
  10   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FOUR(DELCOS,AC,E,CI,CJ,NTERMS,NSAMPS)
C
C    THIS SUBROUTINE CALCULATES THE FOURIER COEFFICIENTS,CI AND CJ,FOR A
C    FIELD PATTERN,E,IN EQUAL STEPS OF COSINE THETA.
C
      DIMENSION AC(100),CI(51),CJ(50),E(100)
      DATA PI/3.1415927/
C---   CALCULATE CI TERMS
      DO 20 I=1,NTERMS+1
      SUM=0.
      N=I-1
      DO 10 J=1,NSAMPS
      ARG=N*PI*AC(J)
   10 SUM=SUM+E(J)*COS(ARG)
20    CI(I)=DELCOS*SUM
      CI(1)=0.5*CI(1)
C---   CALCULATE CJ TERMS
      DO 40 I=1,NTERMS
      SUM=0.
      DO 30 J=1,NSAMPS
      ARG=I*PI*AC(J)
   30 SUM=SUM+E(J)*SIN(ARG)
40    CJ(I)=DELCOS*SUM
      RETURN
      END
```

```
      SUBROUTINE FOURO(DELCOS,E,CI,NSAMPS)
C
C    THIS SUBROUTINE CALCULATES THE FOURIER COEFFICIENT CI(0) FOR A
C    FIELD PATTERN,E,IN EQUAL STEPS OF COSINE THETA.
C
      DIMENSION E(100)
      DATA PI/3.1415927/
C---   CALCULATE CI(0) TERM
      SUM=0.
      DO 10 J=1,NSAMPS
   10 SUM=SUM+E(J)
      CI=0.5*DELCOS*SUM
      RETURN
      END
```

```
      SUBROUTINE MULLER(COE,COEI,N1,ROOTR,ROOTI)
C     FORTRAN              MULLER     C2.2-001B
C MULLER
      DIMENSION COE(51),ROOTR(50),ROOTI(50),COEI(51)
      N2=N1+1
      N4=0
      I=N1+1
   19 IF(COE(I))9,7,9
    7 IF(COEI(I)) 9,1,9
    1 N4=N4+1
      ROOTR(N4)=0.
      ROOTI(N4)=0.
      I=I-1
      IF(N4-N1)19,37,19
    9 CONTINUE
   10 AXR=0.97
      AXI=0.
      N3=1
      ALP1R=AXR
      ALP1I=AXI
      M=1
      GO TO 99
   11 BET1R=TEMR
      BET1I=TEMI
      AXR=0.98
      ALP2R=AXR
      ALP2I=AXI
      M=2
      GO TO 99
   12 BET2R=TEMR
      BET2I=TEMI
      AXR=0.99
      ALP3R=AXR
      ALP3I=AXI
      M=3
      GO TO 99
   13 BET3R=TEMR
      BET3I=TEMI
   14 TE1=ALP1R-ALP3R
      TE2=ALP1I-ALP3I
      TE5=ALP3R-ALP2R
      TE6=ALP3I-ALP2I
      TEM=TE5*TE5+TE6*TE6
      TE3=(TE1*TE5+TE2*TE6)/TEM
      TE4=(TE2*TE5-TE1*TE6)/TEM
      TE7=TE3+1.
      TE9=TE3*TE3-TE4*TE4
      TE10=2.*TE3*TE4
      DE15=TE7*BET3R-TE4*BET3I
      DE16=TE7*BET3I+TE4*BET3R
      TE11=TE3*BET2R-TE4*BET2I+BET1R-DE15
      TE12=TE3*BET2I+TE4*BET2R+BET1I-DE16
      TE7=TE9-1.
      TE1=TE9*BET2R-TE10*BET2I
      TE2=TE9*BET2I+TE10*BET2R
      TE13=TE1-BET1R-TE7*BET3R+TE10*BET3I
```

```
      TE14=TE2-BET1I-TE7*BET3I-TE10*BET3R
      TE15=DE15*TE3-DE16*TE4
      TE16=DE15*TE4+DE16*TE3
      TE1=TE13*TE13-TE14*TE14-4.*(TE11*TE15-TE12*TE16)
      TE2=2.*TE13*TE14-4.*(TE12*TE15+TE11*TE16)
      TEM=SQRT(TE1*TE1+TE2*TE2)
      IF(TE1)113,113,112
  113 TE4=SQRT(.5*(TEM-TE1))
      TE3=.5*TE2/TE4
      GO TO 111
  112 TE3=SQRT(.5*(TEM+TE1))
      IF(TE2)110,200,200
  110 TE3=-TE3
  200 TE4=.5*TE2/TE3
  111 TE7=TE13+TE3
      TE8=TE14+TE4
      TE9=TE13-TE3
      TE10=TE14-TE4
      TE1=2.*TE15
      TE2=2.*TE16
      IF(TE7*TE7+TE8*TE8-TE9*TE9-TE10*TE10)204,204,205
  204 TE7=TE9
      TE8=TE10
  205 TEM=TE7*TE7+TE8*TE8
      TE3=(TE1*TE7+TE2*TE8)/TEM
      TE4=(TE2*TE7-TE1*TE8)/TEM
      AXR=ALP3R+TE3*TE5-TE4*TE6
      AXI=ALP3I+TE3*TE6+TE4*TE5
      ALP4R=AXR
      ALP4I=AXI
      M=4
      GO TO 99
   15 N6=1
   38 IF(ABS(HELL)+ABS(BELL)-1.E-20)18,18,16
   16 TE7=ABS(ALP3R-AXR)+ABS(ALP3I-AXI)
      IF(TE7/(ABS(AXR)+ABS(AXI))-1.E-7)18,18,17
   17 N3=N3+1
      ALP1R=ALP2R
      ALP1I=ALP2I
      ALP2R=ALP3R
      ALP2I=ALP3I
      ALP3R=ALP4R
      ALP3I=ALP4I
      BET1R=BET2R
      BET1I=BET2I
      BET2R=BET3R
      BET2I=BET3I
      BET3R=TEMR
      BET3I=TEMI
      IF(N3 .LT. 100) GO TO 14
      IFLAG=1
      WRITE(3,400)
 400  FORMAT(1X,'*--ERROR-- BAD ROOTS IN SUBROUTINE MULLER*')
   18 N4=N4+1
      ROOTR(N4)=ALP4R
      ROOTI(N4)=ALP4I
      IF(IFLAG .EQ. 1) WRITE(3,401) N4,ROOTR(N4),ROOTI(N4)
```

```
401    FORMAT(1X,I5,2E15.5)
       IFLAG=0
       N3=0
    41 IF(N4-N1)10,37,37
    37 RETURN
    99 TEMR=COE(1)
       TEMI=0.0
       DO 100  I=1,N1
       TE1=TEMR*AXR-TEMI*AXI
       TEMI=TEMI*AXR+TEMR*AXI +COEI(I+1)
   100 TEMR=       TE1+COE(I+1)
       HELL=TEMR
       BELL=TEMI
    42 IF(N4)102,103,102
  102  DO 101  I=1,N4
       TEM1=AXR-ROOTR(I)
       TEM2=AXI-ROOTI(I)
       TE1=TEM1*TEM1+TEM2*TEM2
       TE2=(TEMR*TEM1+TEMI*TEM2)/TE1
       TEMI=(TEMI*TEM1-TEMR*TEM2)/TE1
   101 TEMR=TE2
   103 GO TO(11,12,13,15),M
       END
```

```
      SUBROUTINE PRONY(DELX,YVALS,A,ALPHA,NTERMS,NPOINT)
      DIMENSION RINR(51), RINI(51), ROOTR(50), ROOTI(50)
      COMPLEX YVALS, F,  FBARF, B, FBARB, F1, F1BARY,EIG,
     1    SOLN, A, ALPHA,F1BARF
      COMMON /BLOCK1/ F(100,50),FBARF(50,50),B(100),FBARB(50)
      EQUIVALENCE (F,F1),(FBARF,F1BARF),(FBARB,F1BARY)
      DIMENSION F1(100,50),  F1BARF(50,50), F1BARY(50)
      DIMENSION SOLN(50), A(50), ALPHA(50)
      DIMENSION YVALS(100)
      IDIFF = NPOINT - NTERMS

      IF(IDIFF .LE. 128) GO TO 200
      WRITE(3,430)
430   FORMAT(1X,'*--ERROR--   NPTS-NPOLES .GT. 128*')
200   CONTINUE

COMMENT----NOW TO GENERATE F MATRIX

      DO  800  IA=1, NTERMS
      DO  800  IB=1, IDIFF
      IPASS=NTERMS
  800 F(IB,IA) = YVALS(IA+IB-1)

COMMENT----NOW GENERATE B VECTOR

      DO  801  IE=1, IDIFF
  801 B(IE) = - YVALS(IE+IPASS)


COMMENT----COMPUTE FBARF AND FBARB
      DO 802 I=1,NTERMS
      DO 802 J=1,NTERMS
      FBARF(I,J)=0.
      DO 802 K=1,IDIFF
802     FBARF(I,J)=FBARF(I,J)+F(K,I)*F(K,J)

      DO 803 I=1,NTERMS
      FBARB(I)=0.
      DO 803 K=1,IDIFF
803     FBARB(I)=FBARB(I)+F(K,I)*B(K)

COMMENT----USE CROUT TO SOLVE EQN FBARF*S=FBARB FOR S

      CALL CROUT(FBARF,SOLN, FBARB, NTERMS, 1)

COMMENT----CIN NOW CONTAINS SOLUTIONS FOR S
COMMENT----NOW TO FIND ROOTS OF POLYNOMIAL WITH S VECTOR AS COEFFS

      RINR(1)=1.
      RINI(1)=0.
      DO  804  IM=1,NTERMS
      IZ=NTERMS+1-IM
      IY=IM+1
      RINR(IY) = REAL(SOLN(IZ))
 804  RINI(IY) = AIMAG(SOLN(IZ))
      CALL MULLER (RINR, RINI, NTERMS, ROOTR, ROOTI)
```

```
COMMENT----NOW USE LOG (COMPLEX) TO FIND T1 AND T2, ETC.

      DO  805  IQ=1,NTERMS
  805 ALPHA(IQ) = CLOG(CMPLX(ROOTR(IQ),ROOTI(IQ)))/DELX


COMMENT----NOW TO DO A LEAST SQUARES FIT FOR THE A COEFFS IN A*EXP(ALPHA

COMMENT----FIRST GENERATE THE F1 MATRIX

      DO  806  JA=1,NTERMS
      F1(1,JA)=1.
      F1(2,JA) = CMPLX(ROOTR(JA), ROOTI(JA))
      DO  806  JB=3,NPOINT
  806 F1(JB,JA) = F1(JB-1,JA) * F1(2,JA)

COMMENT----NOW FIND F1BAR


COMMENT----COMPUTE F1BARF AND F1BARY


      DO 807 I=1,NTERMS
      DO 807 J=1,NTERMS
      F1BARF(I,J)=0.
      DO 807 K=1,NPOINT
807    F1BARF(I,J)=F1BARF(I,J)+F1(K,I)*F1(K,J)

      DO 808 I=1,NTERMS
      F1BARY(I)=0.
      DO 808 K=1,NPOINT
808    F1BARY(I)=F1BARY(I)+F1(K,I)*YVALS(K)


COMMENT----USE CROUT TO SOLVE EQN F1BARF*A=F1BARY  FOR A


      CALL CROUT (F1BARF, A, F1BARY, NTERMS, 1)


      RETURN
      END
```